CIRCULAR MOTION

IB PHYSICS | COMPLETED NOTES

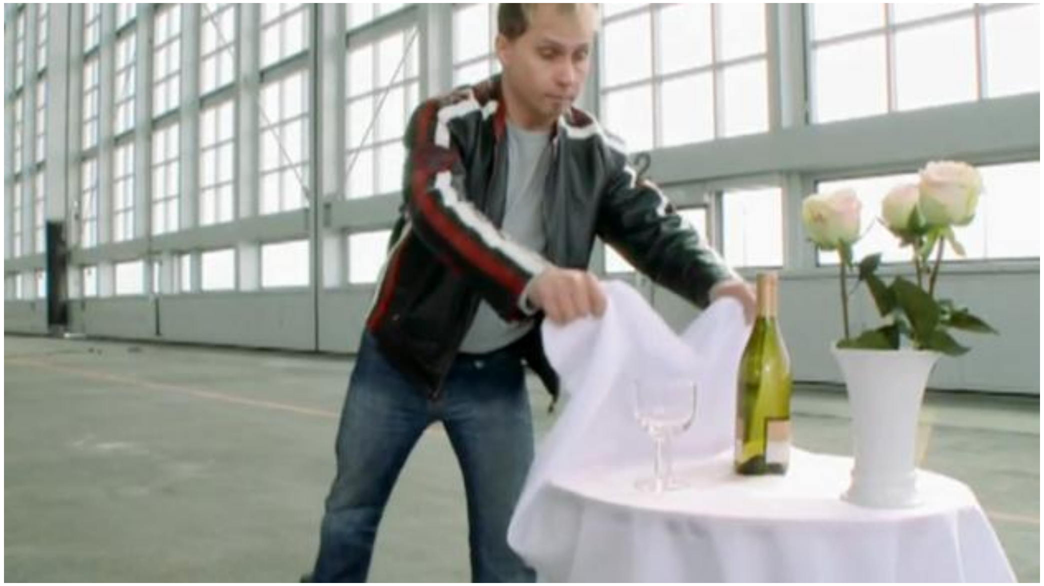
Defining Circular Motion

IB PHYSICS | CIRCULAR MOTION

Remember Newton's 1st?

A body will remain at rest or moving with constant velocity unless acted upon by an unbalanced force

"Law of Inertia"



Try This…

I’m usually running late for school and sometimes I forget my plate of pop tarts on the top of my car. What happens when I take a sharp turn to the right? Why?

Pop Tarts will keep moving forward (in a straight line) unless an outside force acts upon them

Remember back…

There are 3 ways that an object can be experiencing acceleration?

Speeding Up

Slowing Down

Changing Direction

You already know some of this...

If each blade in the wind farm animation is 30 meters long, estimate the speed (in m s^{-1}) of the tip of one turbine blade.

Distance travelled by the tip of the blade for one revolution:

$$d = 2\pi r = 2\pi(30) = \mathbf{188.5\ m}$$

Time for one revolution = **2.9 seconds**

$$v = \frac{d}{t} = \frac{188.5\ \text{m}}{2.9\ \text{s}} = \mathbf{65.0\ m\ s^{-1}}$$

Think about the Circle…

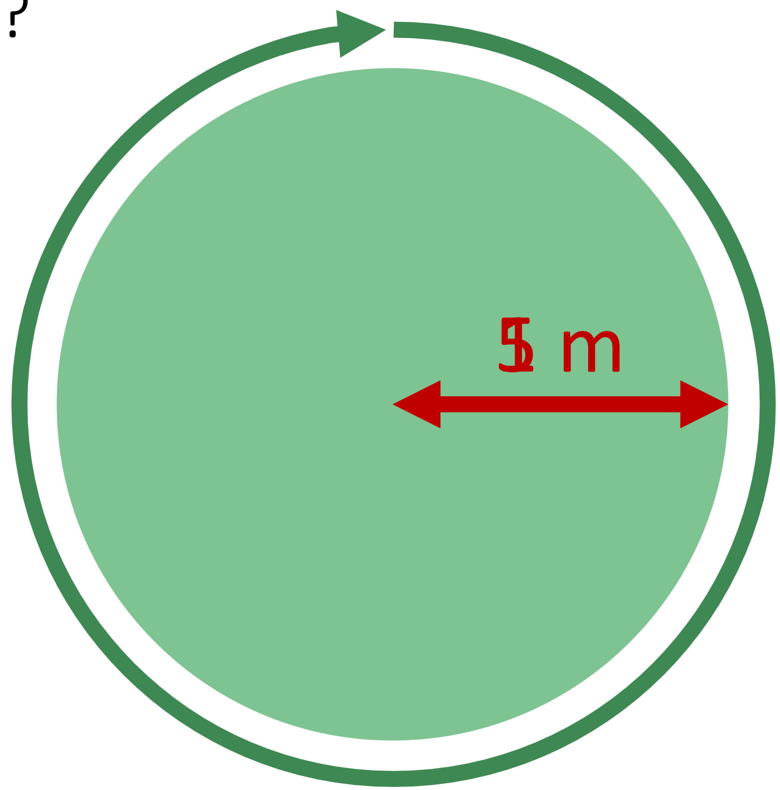
If you walked around this circle once, what is your total distance?

$C = 2\pi r = 2\pi(1\text{ m})$

$C = 2\pi r = 2\pi(5\text{ m})$

$C = 6.28$ meters

$C = 31.4$ meters



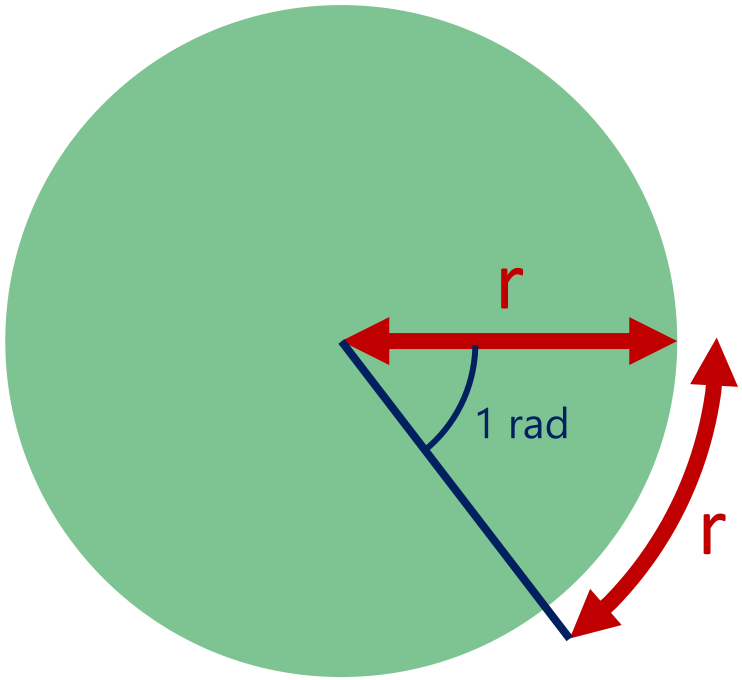
What is a Radian??

We can define a circular distance in terms of a generic radius, r…

$$C = 2\pi r$$

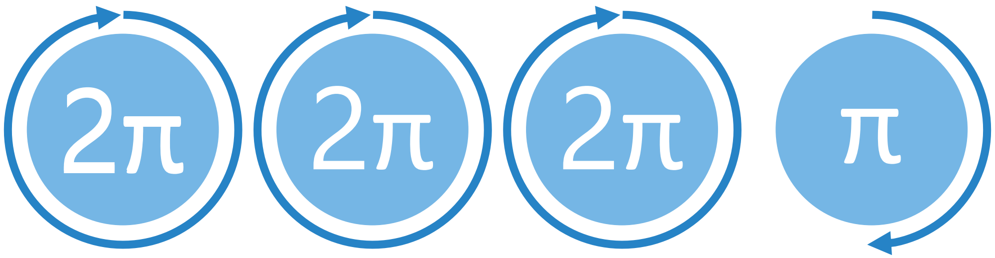
How many radians are there in one full revolution?

2π radians



Try this….

If a child on a merry-go-round rotates 3.5 times, what is their **angular distance** in radians?



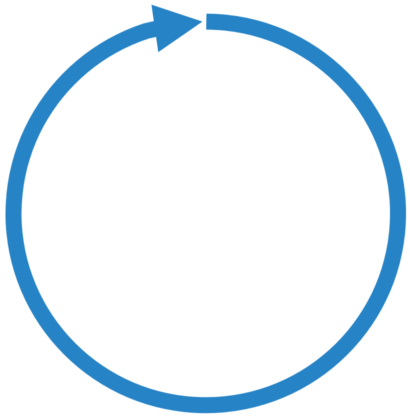
$$3.5(2\pi) = 7\pi$$
$$= 22 \text{ rad}$$

If an ant on a record player spins for an angular displacement of 14 radians, how many revolutions has it experienced?

$$\frac{14}{2\pi} = 2.23\ revolutions$$

Timing Circular Motion

Period	**T**	**[s]**
Time for complete revolution		



Angular Velocity

For Linear Motion:

$$v = \frac{distance}{time} = \frac{d}{t}$$

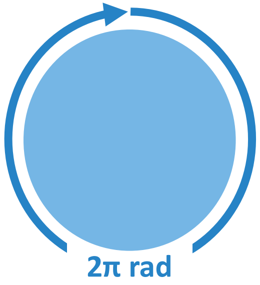
For Circular Motion:

$$\omega = \frac{angular\ distance}{time}$$

Angular Velocity [rad s^{-1}] → ω

If you have a single revolution:

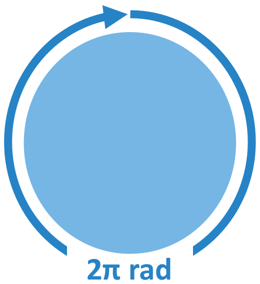
$$\omega = \frac{angular\ distance}{time} = \frac{2\pi}{T}$$



Time for one revolution = T

Try this....

A ferris wheel takes 40 seconds (T) to make on full revolution, what is its angular velocity in rad/s?



$$\omega = \frac{2\pi}{T} = \frac{2\pi}{40} = 0.157\, rad\, s^{-1}$$

A car tire rotates with an average angular velocity of 29 rad/s. In what time interval will the tire rotate 3.5 times?

$$\omega = \frac{angular\ distance}{time} \qquad 29\,\frac{rad}{s} = \frac{3.5(2\pi)}{t} \qquad t = 0.758\ \mathrm{s}$$

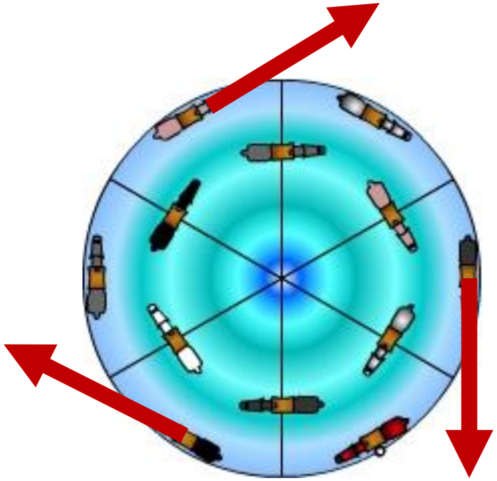
$$\omega = \frac{2\pi}{T} \qquad 29\,\frac{rad}{s} = \frac{2\pi}{T} \qquad T = 0.217\ \mathrm{s} \qquad 0.217 \times 3.5 = \mathbf{0.758\ s}$$

Time for one revolution

Linear Velocity

At any given point, an object with circular motion will also have an instantaneous linear velocity.

This velocity will be in the direction _tangent_ to the curve



Calculating Linear Velocity

$$v = \frac{d}{t}$$

$$v = \frac{2\pi r}{T}$$

Distance of one complete rotation (circumference)

Time for one complete rotation (Period)

Calculating Linear Velocity

$$v = \frac{2\pi r}{T}$$

$$\omega = \frac{2\pi}{T}$$

$$v = \omega r$$

IB Physics Data Booklet

Sub-topic 6.1 – Circular motion

$$v = \omega r$$

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

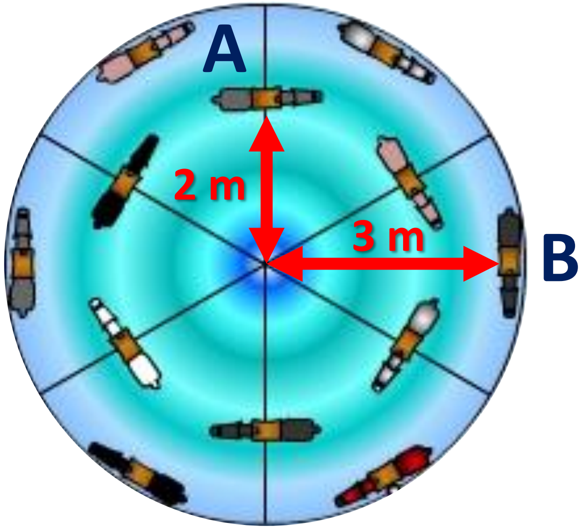
$$F = \frac{mv^2}{r} = m\omega^2 r$$

$$\omega = \frac{angular\ distance}{time}$$

For one revolution:

$$\omega = \frac{2\pi}{T}$$

Try this….



If the carousel spins at 1 complete rotation every 10 seconds, what is the angular and linear velocity of each row?

Time for 1 Rotation:

T = 10 s

A	$\omega = \frac{2\pi}{T}$ $\omega = \frac{2\pi}{10}$ $= 0.63\ rad\ s^{-1}$	$v = \omega r$ $v = (0.63)(2)$ $= 1.3\ m\ s^{-1}$
B	$\omega = \frac{2\pi}{T}$ $\omega = \frac{2\pi}{10}$ $= 0.63\ rad\ s^{-1}$	$v = \omega r$ $v = (0.63)(3)$ $= 1.9\ m\ s^{-1}$

Try this....

If you were sitting 4 m from the center of a carousel spinning at 12 rad s^{-1} and threw a ball in the air, how fast would the ball continue in a straight line?

$r = 4$ m

$\omega = 12$ rad s^{-1}

$v = \omega r = (12)(4) = \mathbf{48\ m\ s^{-1}}$

A woman passes through a revolving door with a tangential speed of 1.8 m s^{-1}. If she is 0.8 m from the center of the door, what is the door's angular velocity?

$v = 1.8$ m s^{-1}

$r = 0.8$ m

$v = \omega r$

$1.8 = \omega(0.8)$

$\boldsymbol{\omega} = \mathbf{2.25\ rad\ s^{-1}}$

Lesson Takeaways

- ❑ I can convert between angular displacement in revolutions and radians
- ❑ I can define and measure the **period** of circular motion
- ❑ I can calculate angular velocity in rad/s
- ❑ I can describe and calculate tangential velocity based on the angular velocity and radius

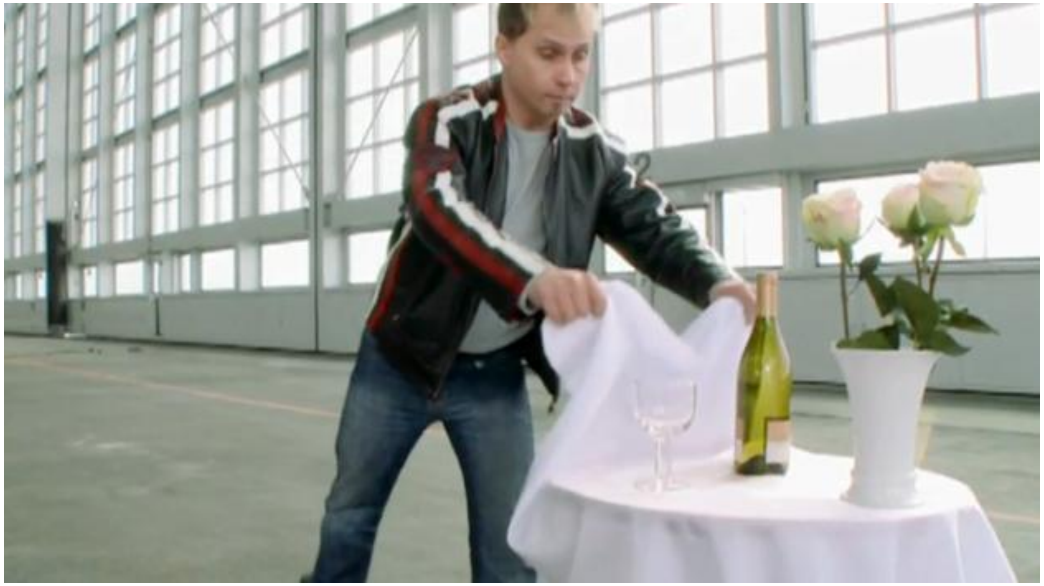
Centripetal Force and Acceleration

IB PHYSICS | CIRCULAR MOTION

Remember Newton's 1st?

A body will remain at rest or moving with constant velocity unless acted upon by an unbalanced force

"Law of Inertia"



Remember back…

There are 3 ways that an object can be experiencing acceleration?

Speeding Up

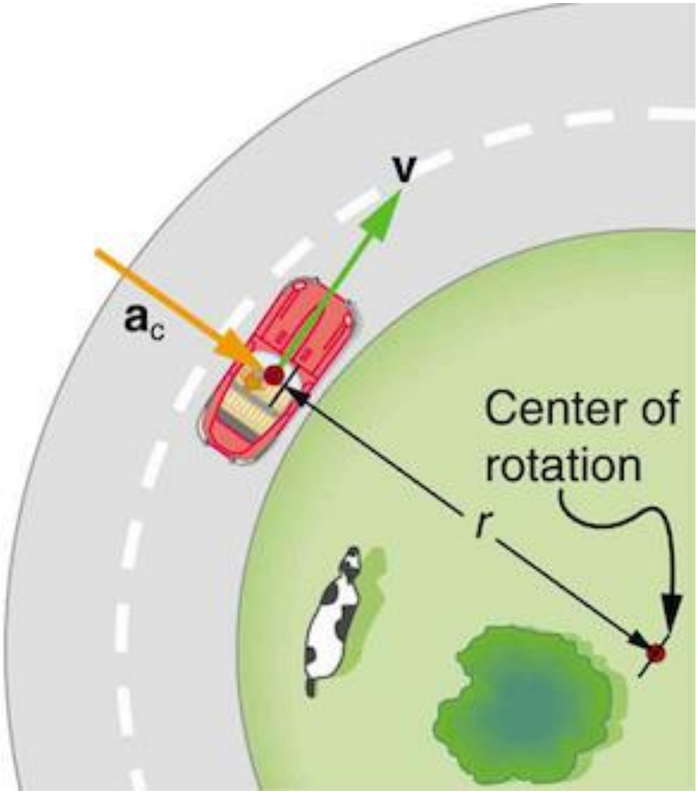
Slowing Down

Changing Direction

Centripetal Acceleration

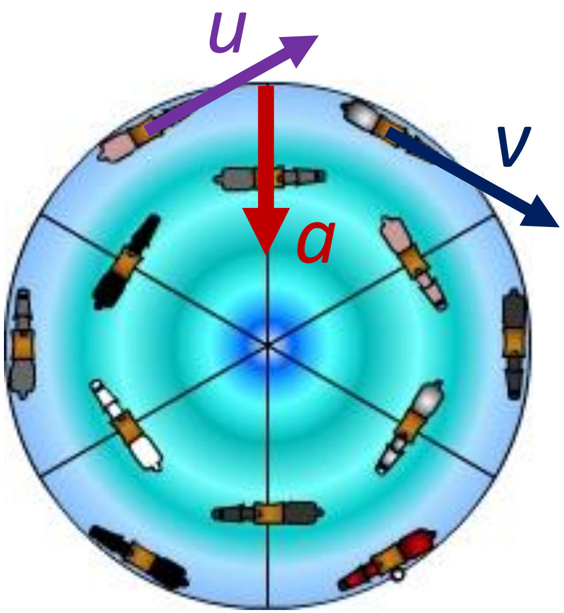
Centripetal acceleration represents the rate of change of velocity and its direction

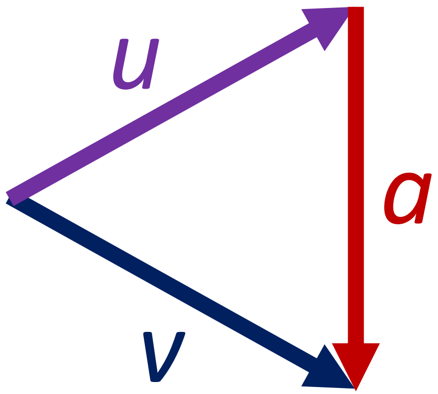
$$a = \frac{v^2}{r}$$



Centripetal Acceleration

Centripetal acceleration can be seen when finding the change between velocity vectors





Centripetal acceleration will always point to the **center**

Calculating Centripetal Acceleration

$$a = \frac{v^2}{r}$$

$$v = \omega r$$

$$\omega = \frac{2\pi}{T}$$

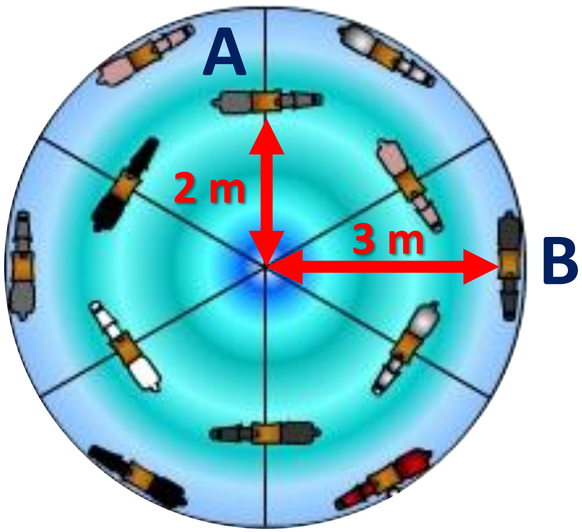
$$v = \frac{2\pi r}{T}$$

$$a = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\frac{4\pi^2 r^2}{T^2}}{r} = \frac{4\pi^2 r^2}{T^2 r} = \frac{\mathbf{4\pi^2 r}}{\mathbf{T^2}}$$

IB Physics Data Booklet

Sub-topic 6.1 – Circular motion
$v = \omega r$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ $F = \frac{mv^2}{r} = m\omega^2 r$

Try this….



If the carousel spins at 1 complete rotation every 10 seconds, what is the centripetal acceleration for each row?

A	$\omega = 0.63$ rad s^{-1} | $v = 1.3$ m s^{-1} $a = \frac{1.3^2}{2} = 0.843 \text{ m s}^{-2}$
B	$\omega = 0.63$ rad s^{-1} | $v = 1.9$ m s^{-1} $a = \frac{1.9^2}{3} = 1.20 \text{ m s}^{-2}$

$$a = \frac{v^2}{r}$$

Wait… Where's the Force?

We know from Newton's 2nd Law that every time that we have acceleration, there must be a force causing that change in velocity

$$F = ma$$

$$a = \frac{v^2}{r}$$

$$F = \frac{mv^2}{r}$$

Calculating Centripetal Force

$$F = \frac{mv^2}{r}$$

$$v = \omega r$$

$$F = \frac{m(\omega r)^2}{r} = \boldsymbol{m\omega^2 r}$$

IB Physics Data Booklet

Sub-topic 6.1 – Circular motion
$v = \omega r$
$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$
$F = \frac{mv^2}{r} = m\omega^2 r$

F = ma

Try This...

A **3 kg** rock swings in a circle of radius **5 m**. If its constant speed is **8 m s^{-1}**, what is the centripetal acceleration and force?

$v = \omega r$
$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$
$F = \frac{mv^2}{r} = m\omega^2 r$

m = 3 kg   r = 5 m   v = 8 m s^{-1}

$$a = \frac{v^2}{r} = \frac{8^2}{5} = \mathbf{12.8\ m\ s^{-2}}$$

$$F = ma = (3)(12.8) = \mathbf{38.4\ N}$$

Try This...

A pilot is flying a small plane at 30.0 m s^{-1} with a radius of 100.0 m. If a force of 635 N is needed to maintain the pilot's circular motion, what is the pilot's mass?

$$v = \omega r$$

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$F = \frac{mv^2}{r} = m\omega^2 r$$

v	30 m s^{-1}
r	100 m
F	635 N
m	?

$$F = \frac{mv^2}{r}$$

$$635 = \frac{m(30)^2}{100}$$

$$m = 70.56 \text{ kg}$$

Equation Summary

Sub-topic 6.1 – Circular motion
$v = \omega r$
$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$
$F = \frac{mv^2}{r} = m\omega^2 r$

Velocity

Linear: v → m s^{-1}

Angular: ω → rad s^{-1}

Centripetal Acceleration
changes direction toward center

a_c → m s^{-2}

Centripetal Force
directed toward center

$F = ma$

See derived equations

Lesson Takeaways

- ❑ I can determine the direction and magnitude of centripetal acceleration and centripetal force
- ❑ I can identify circular motion properties in a description and choose an appropriate equation to relate them

Vertical Circular Motion with Tension

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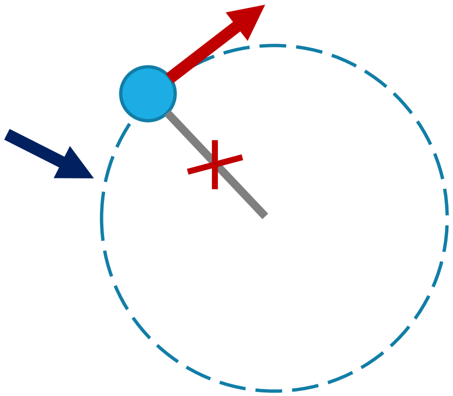
IB Physics Data Booklet

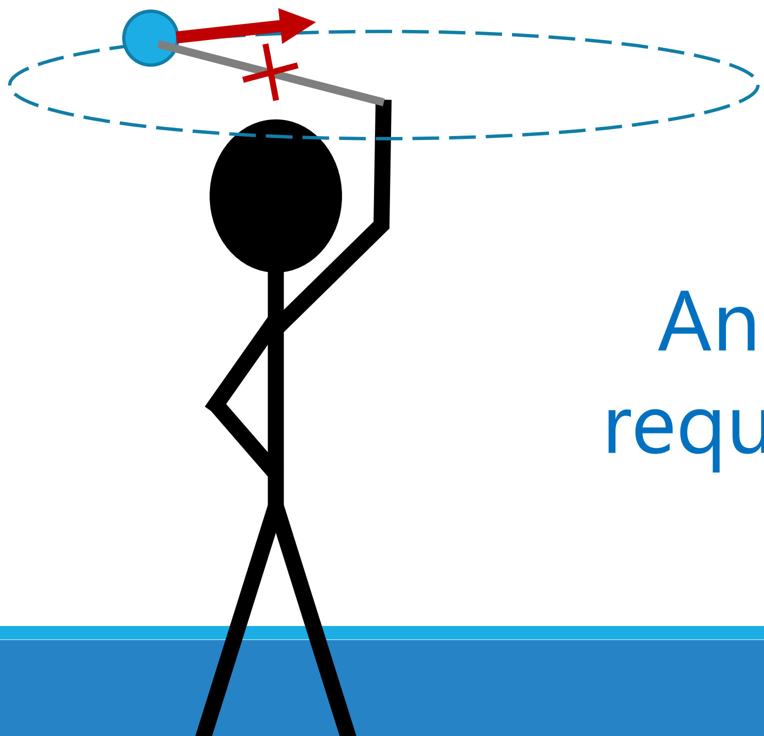
Sub-topic 6.1 – Circular motion	
$v = \omega r$	v – linear velocity (m s^{-1})
$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$	ω – angular velocity (rad s^{-1})
$F = \frac{mv^2}{r} = m\omega^2 r$	r – radius (m)
	T – period (s)
	a – centripetal acceleration (m s^{-2})
	F – centripetal force (N)

Try This…

Top View



If you swing a ball on a string above your head, and the string breaks, what happens?

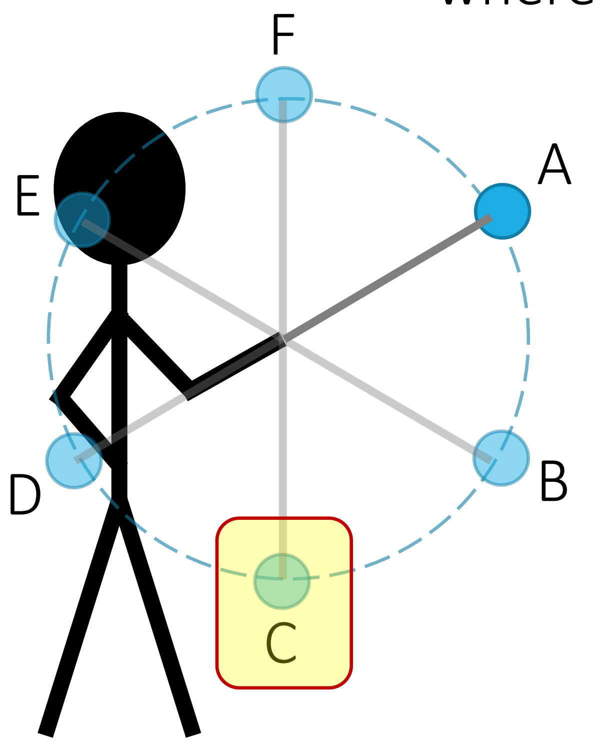


Travels in a straight line tangent to the circle

An inward facing force is required for circular motion

Think about it...

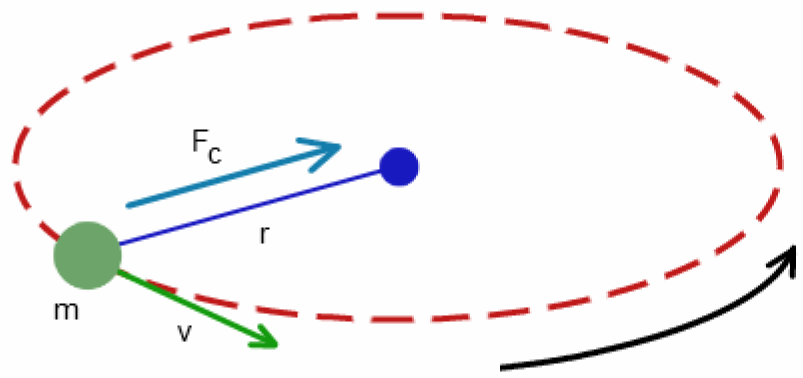
If you swing a ball on a string in a vertical circle, where is the string most likely to break? Why?



Because gravity is pulling against the string at this point

Centripetal Force

Remember, for an object to follow a curved path, there must be an inward pointing centripetal force (F_c)



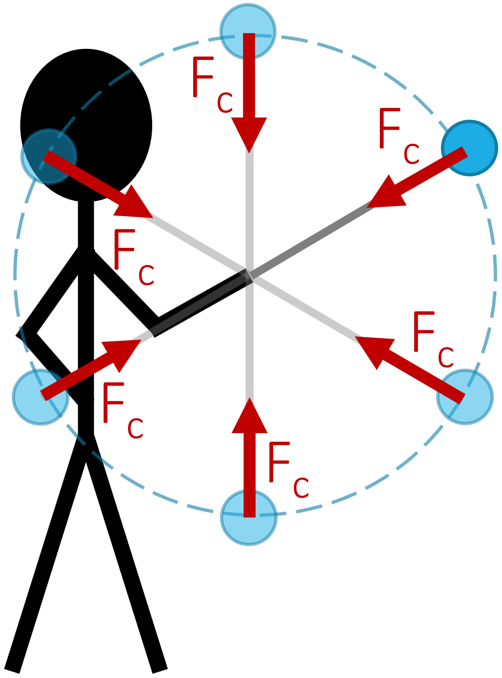
This is not really a force that shows up on a free body diagram like F_g , R, F_f , and F_T .

Rather, it is more like the net force that is required to create that circular motion

If an object is in circular motion: $$F_{net} = F_c$$

Vertical Circle

When you make a vertical circle the net force at all points must equal the centripetal force (F_c)

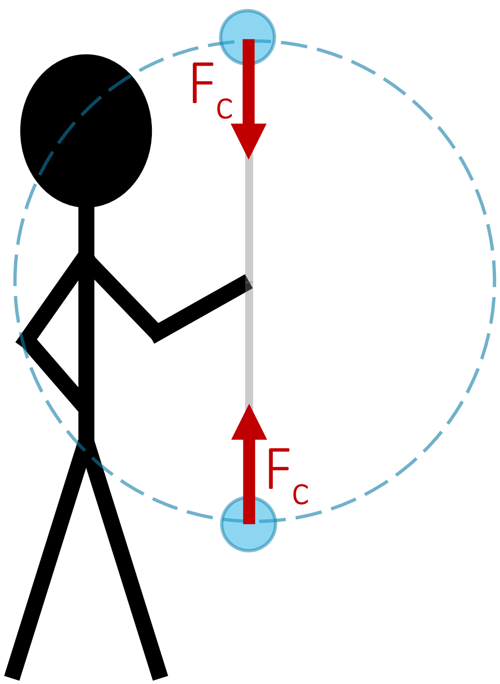


This is the case for horizontal circles too! The main difference is that now the weight is a factor…

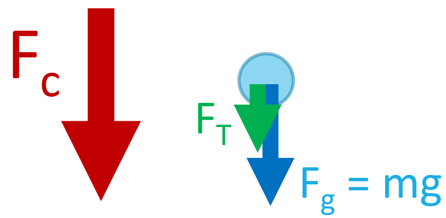
Again, this isn't some magical new force but rather a combination of all forces resulting in…

$$F_{net} = F_c$$

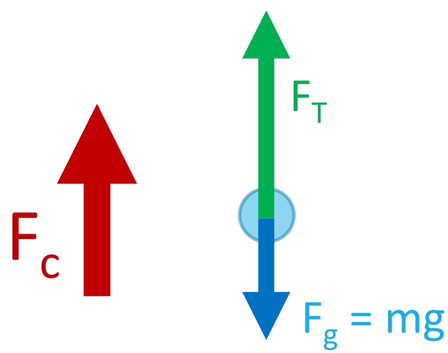
Let’s focus on the top and bottom…



At the Top:



At the Bottom:

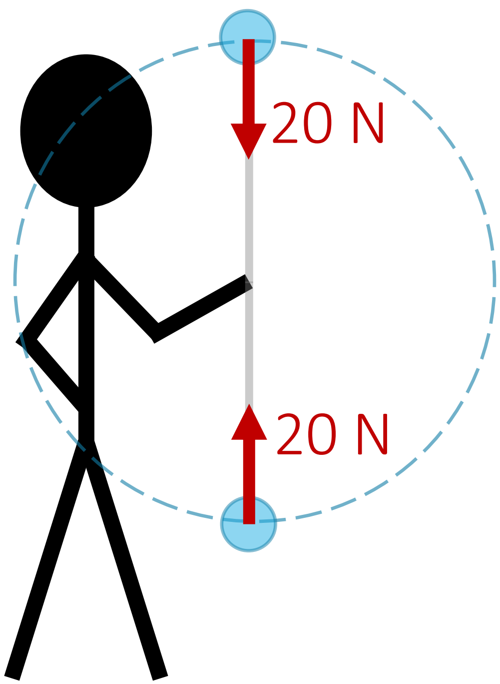


Now with numbers!

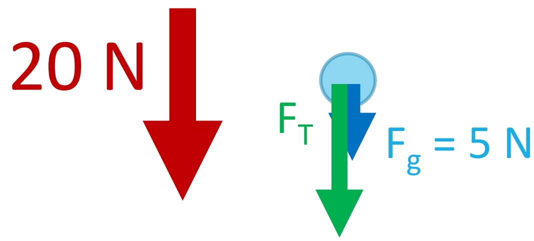
F_c required is 20 N

F_g of object is 5 N

*F_T is determined by comparing the known forces (F_g) to the net force (F_c) and finding the difference

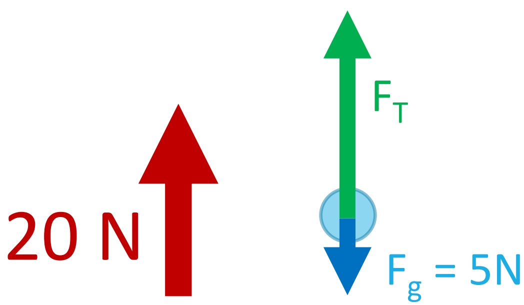


At the Top:



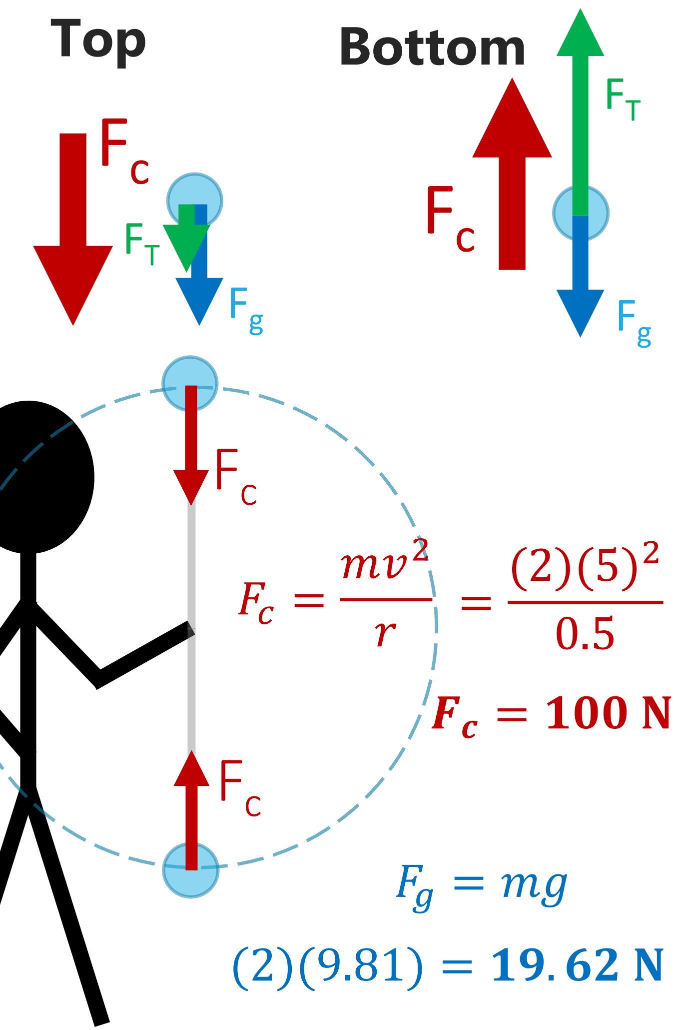
$F_T = 15\text{ N}$

At the Bottom:



$F_T = 25\text{ N}$

What is the tension?



Top

m	2 kg
v_t	5 m/s
r	0.5 m
F_c	100 N
F_{net}	100 N
F_g	19.62 N
F_T	80.38 N

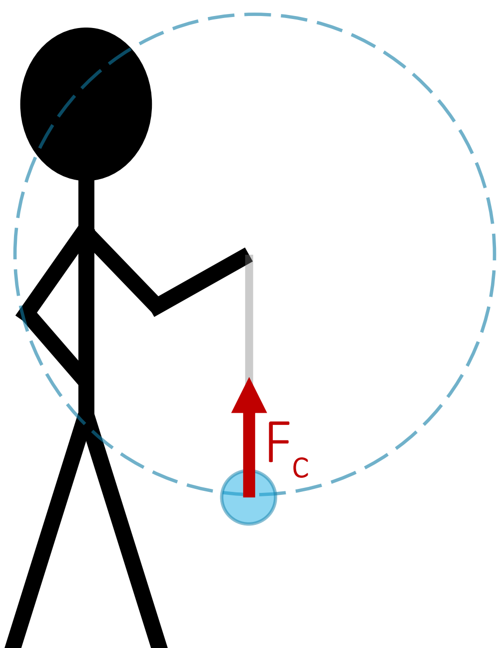
Bottom

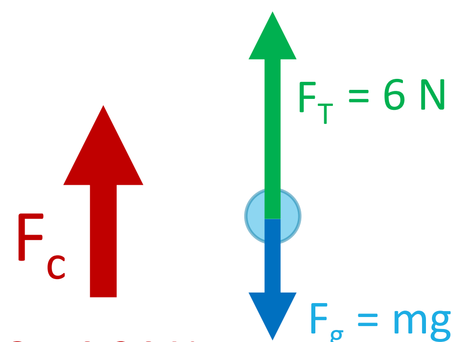
m	2 kg
v_t	5 m/s
r	0.5 m
F_c	100 N
F_{net}	100 N
F_g	19.62 N
F_T	119.62 N

What is the tension?

What is the angular velocity in rad s^{-1} at the bottom of a vertical circle created when a 0.2-kg phone charger is swung with a 0.8 m cord and a tension of 6 N at the lowest point?

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$





$F_g = mg = (0.2)(9.81) = 1.96$ N

$F_c = 6 - 1.96 = 4.04$ N

$$F_c = m\omega^2 r$$

$$4.04 = (0.2)\omega^2(0.8)$$

$$\omega^2 = 25.25$$

$$\boldsymbol{\omega = 5.02\ \mathrm{rad\ s^{-1}}}$$

Lesson Takeaways

- ❑ I can compare the forces on an object at different positions in vertical circular motion
- ❑ I can determine the magnitude and direction of the forces needed for the overall centripetal force
- ❑ I can qualitatively describe how tension changes in a vertical circle

Vertical Circular Motion with a Surface

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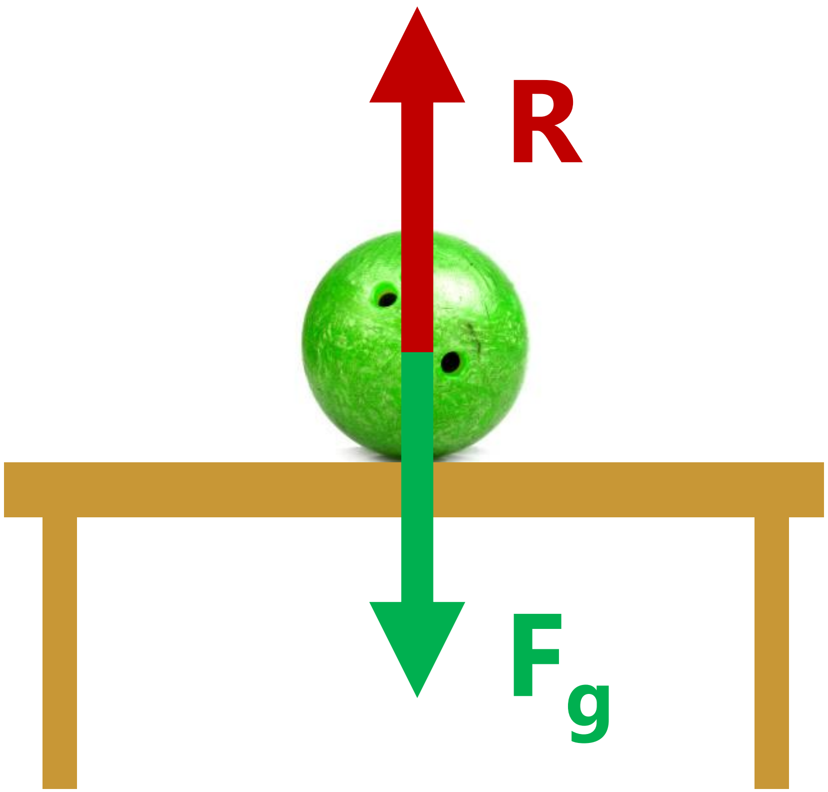
IB Physics Data Booklet

Sub-topic 6.1 – Circular motion	
$v = \omega r$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ $F = \frac{mv^2}{r} = m\omega^2 r$	v – linear velocity (m s^{-1}) ω – angular velocity (rad s^{-1}) r – radius (m) T – period (s) a – centripetal acceleration (m s^{-2}) F – centripetal force (N)

Remember Normal Reaction Force?

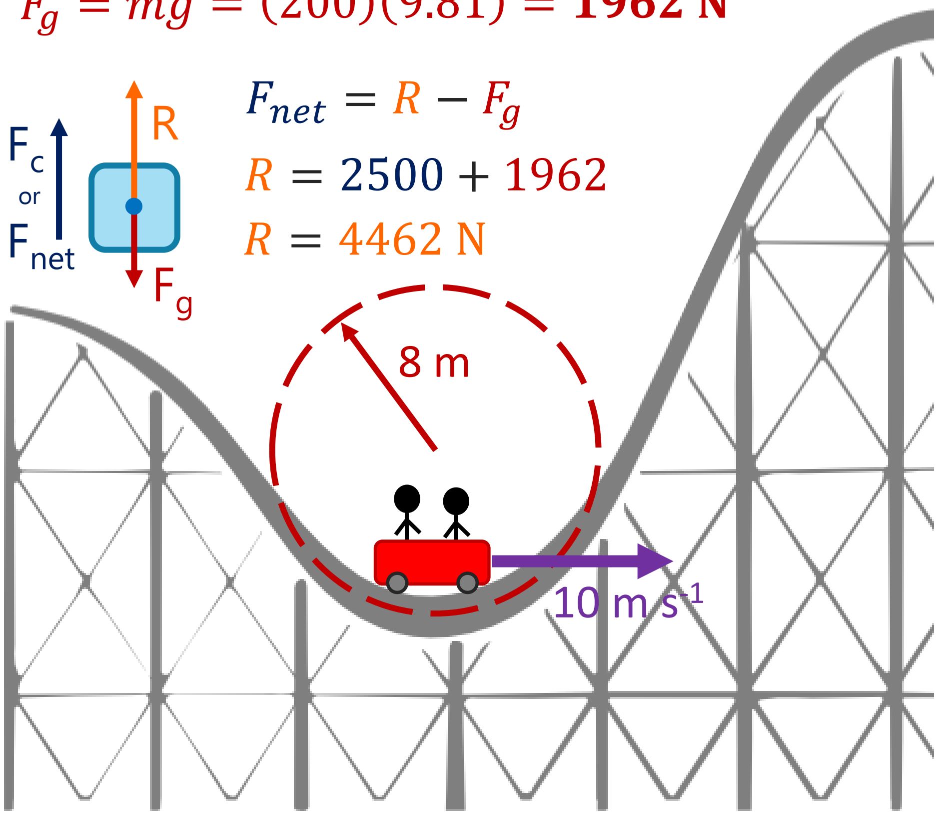
*Always perpendicular to the surface applying the force



Roller Coaster | Bottom

$$F_c = \frac{mv^2}{r} = \frac{(200)(10)^2}{8} = \mathbf{2500\ N}$$

$$F_g = mg = (200)(9.81) = \mathbf{1962\ N}$$



$$F_{net} = R - F_g$$

$$R = 2500 + 1962$$

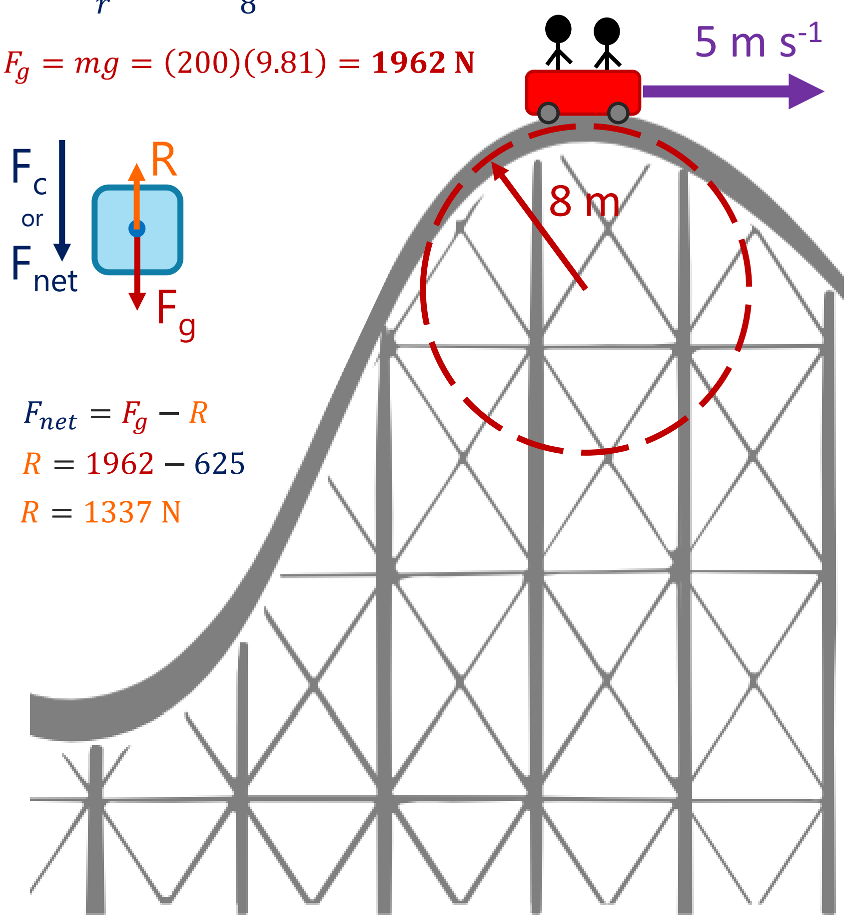
$$R = 4462\ \text{N}$$

m	200 kg
v	10 m s^{-1}
r	8 m
F_c	2500 N
F_{net}	2500 N
F_g	1962 N
R	4462 N

Roller Coaster | Top

$$F_c = \frac{mv^2}{r} = \frac{(200)(5)^2}{8} = \mathbf{625\ N}$$

$$F_g = mg = (200)(9.81) = \mathbf{1962\ N}$$



$$F_{net} = F_g - R$$

$$R = 1962 - 625$$

$$R = 1337\ \text{N}$$

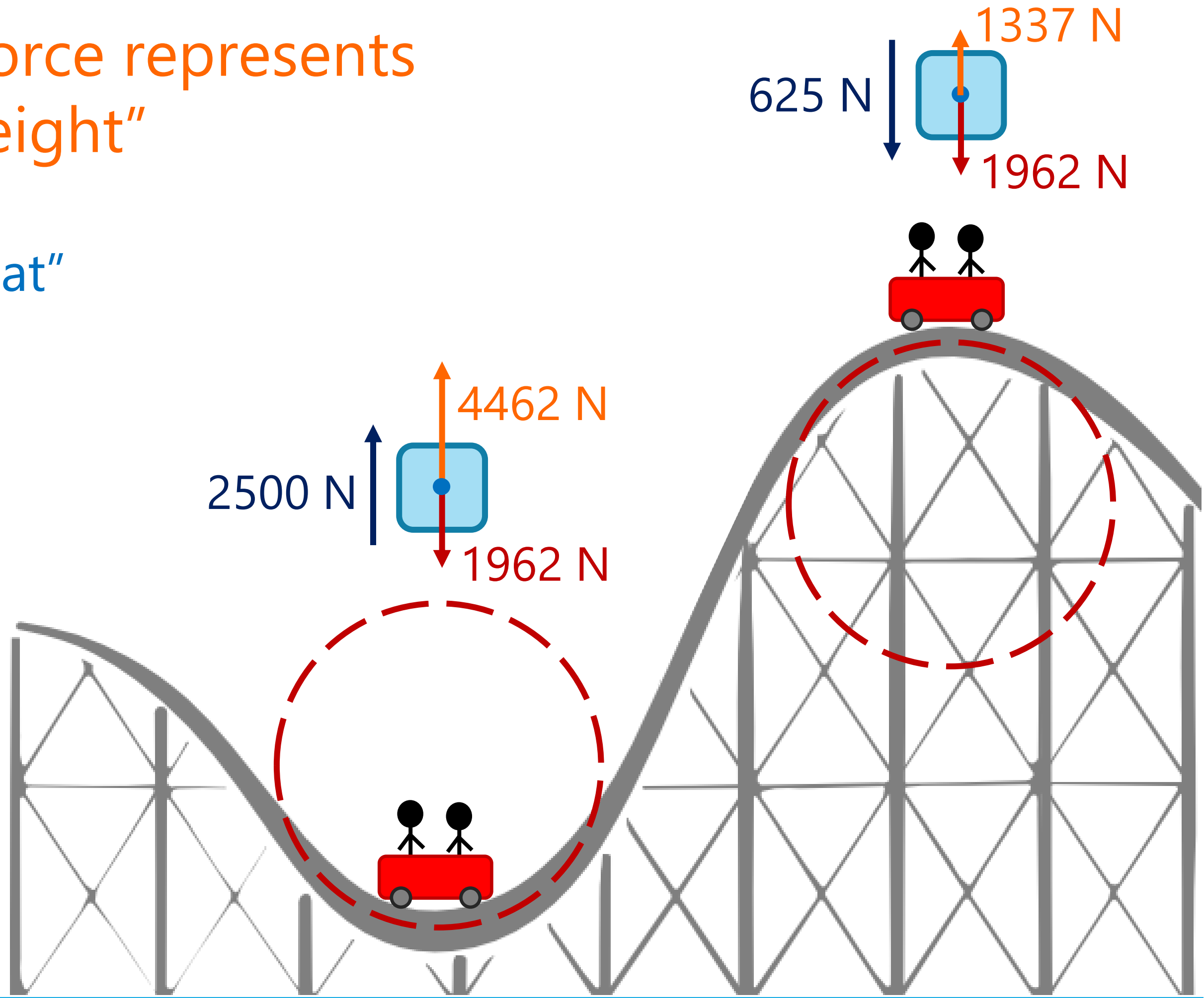
m	200 kg
v_t	5 m s^{-1}
r	8 m
F_c	625 N
F_{net}	625 N
F_g	1962 N
R	1337 N

Perceived Weight

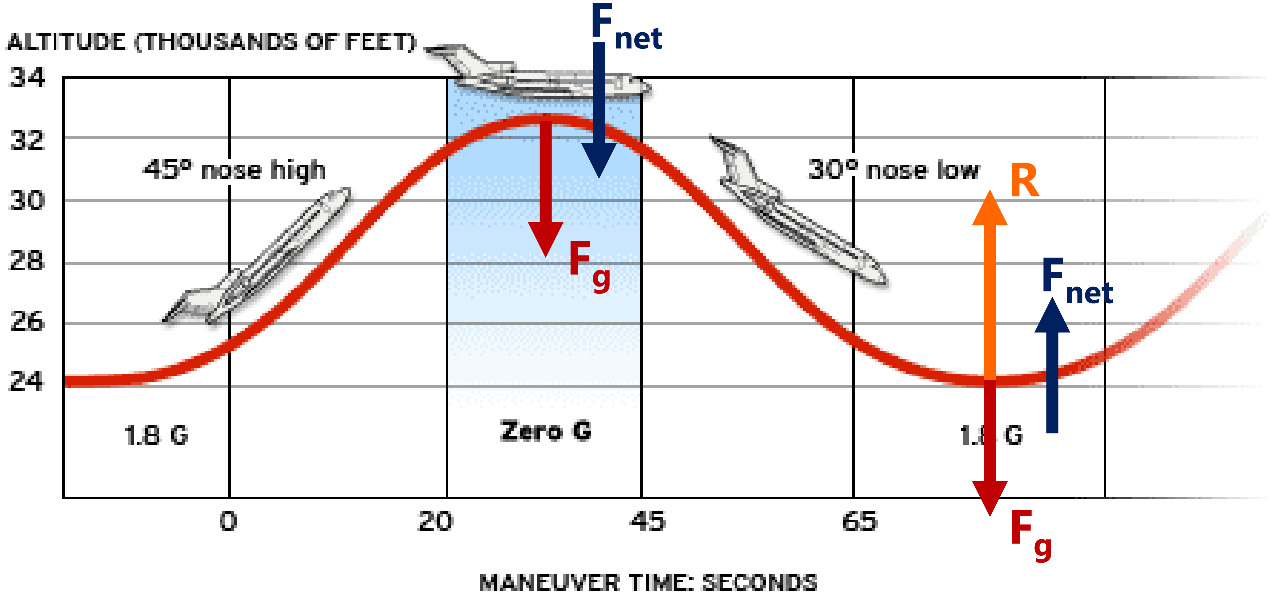
The normal reaction force represents a rider's "perceived weight"

$R > F_g$ | "Squished into seat"

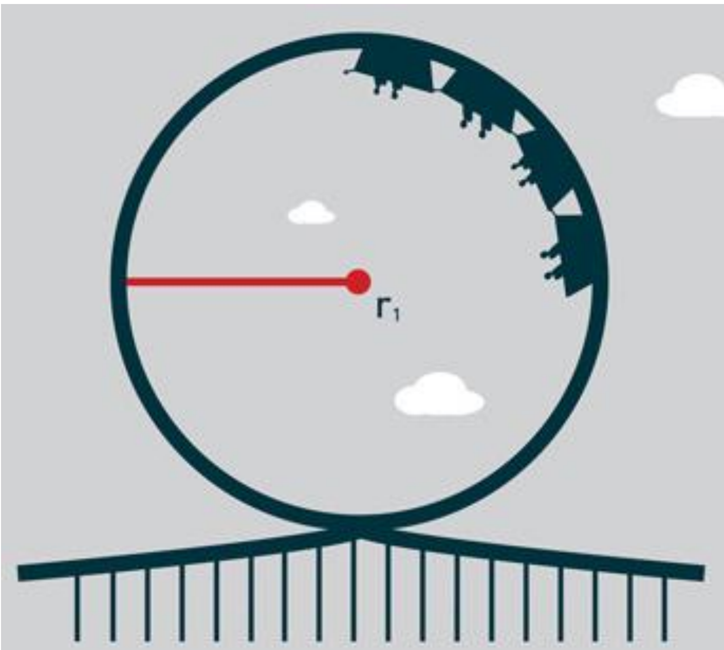
$R < F_g$ | "Weightless"



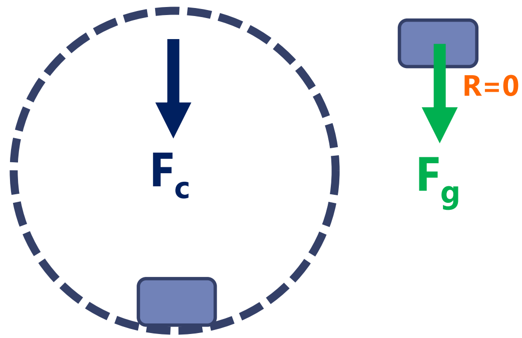
The ultimate "weightless" experience



Loop the Loop!



The velocity needs to be fast enough that the R is greater than 0 N



$$F_c = F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$v = \sqrt{gr}$$

Minimum velocity required = $\sqrt{gr}$

Lesson Takeaways

- ❑ I can compare the forces on an object at different positions in vertical circular motion
- ❑ I can determine the magnitude and direction of the forces needed for the overall centripetal force
- ❑ I can qualitatively describe how normal reaction force changes in a vertical circle
- ❑ I can describe the experience of “weightlessness” in terms of normal reaction force

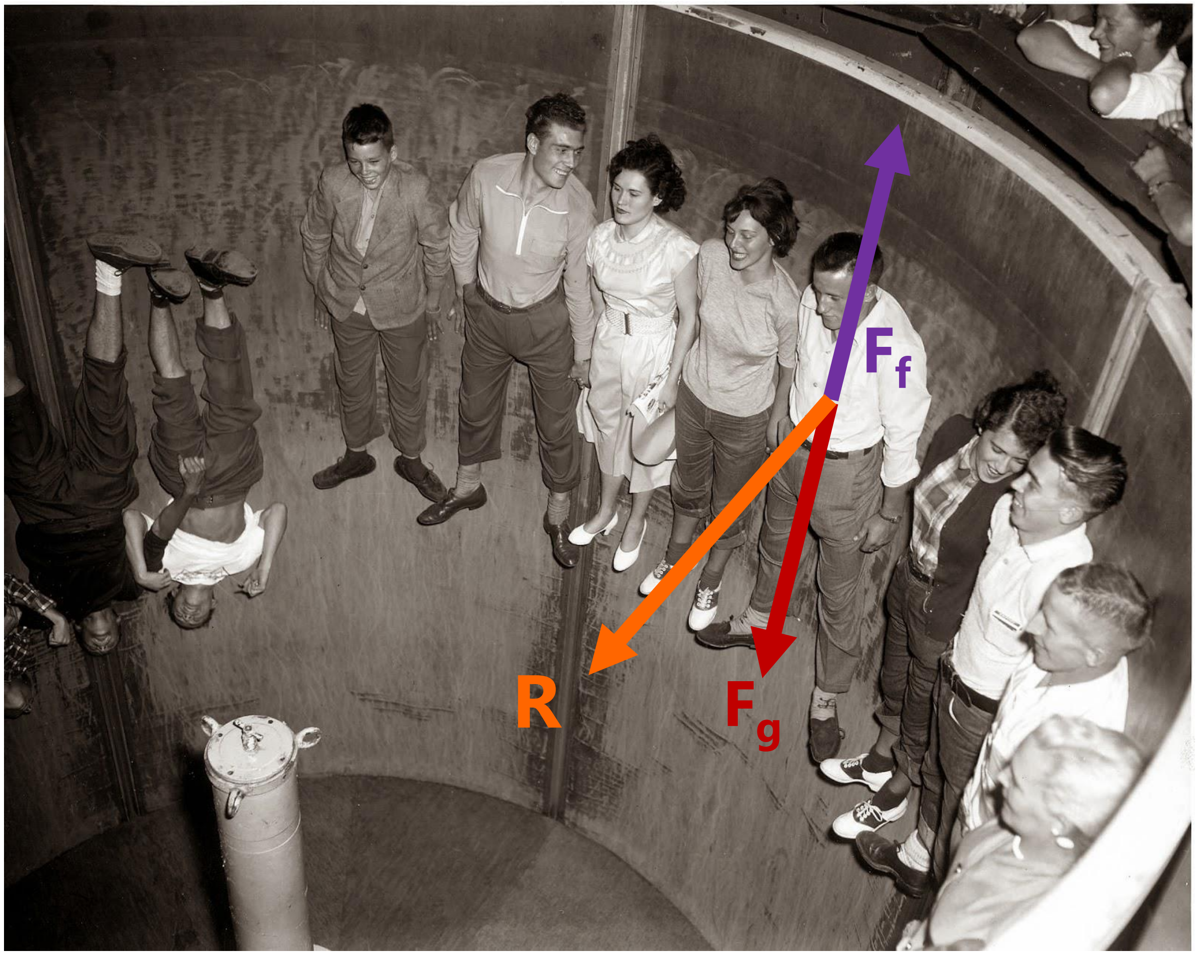
Circular Motion Scenarios
The Rotor

IB PHYSICS | CIRCULAR MOTION

IB Physics Data Booklet

Sub-topic 6.1 – Circular motion	
$v = \omega r$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ $F = \frac{mv^2}{r} = m\omega^2 r$	v – linear velocity (m s^{-1}) ω – angular velocity (rad s^{-1}) r – radius (m) T – period (s) a – centripetal acceleration (m s^{-2}) F – centripetal force (N)

"The Rotor"



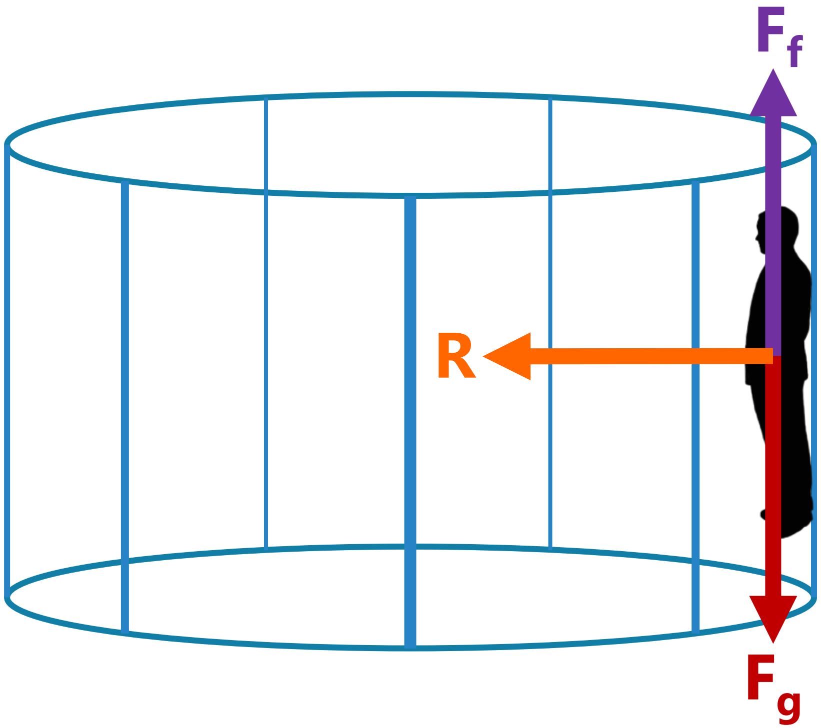
$F_f = \mu R$

$F_g = mg$

Remember "The Rotor"

We can use this example to discuss that there must be an inward force (centripetal force) acting towards the center. But why don't they fall down?!?

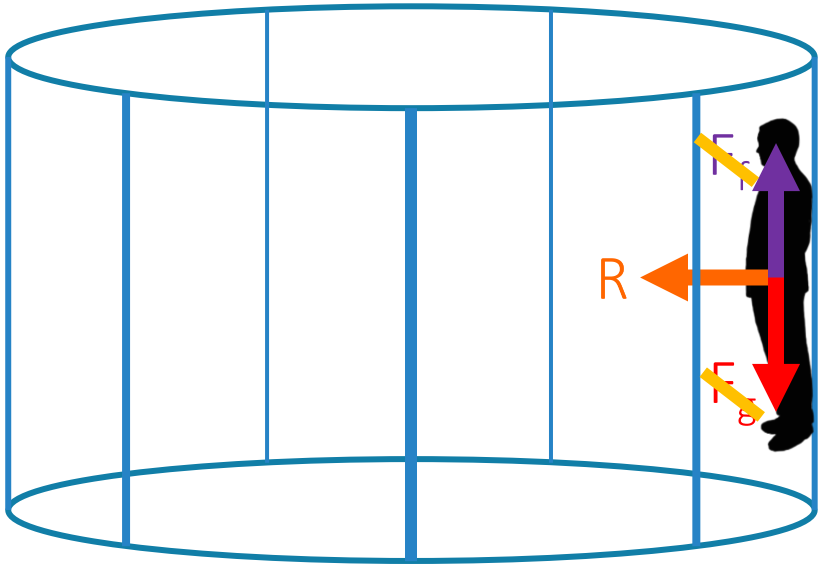
Friction



You can't forget about Friction!

Remember that friction is related to the **normal force** (R) and the **coefficient of friction** (μ) (μ).
The only thing that is different here is that the **normal force** is the **centripetal force**.

$$F_{net} = F_c = R$$



$$F_f = F_g$$

$$F_f = \mu R$$

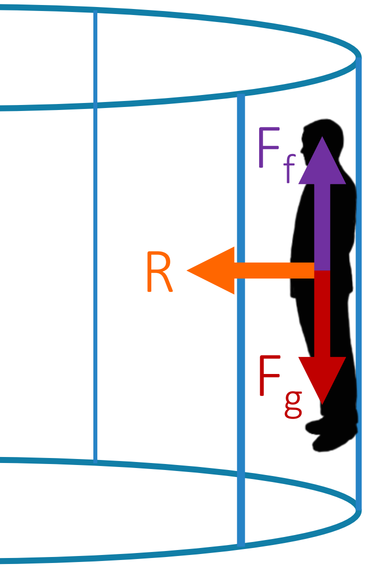
$$F_{net} = R$$

$$F_c = R$$

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Give it a Shot!

The "Rotor" ride is the one which presses you against the walls of the spinning rotor as the floor drops away. The coefficient of static friction between the wall and the 75-kg rider is μ = 0.06. If the ride is rotating at an angular velocity of 5.2 rad s^{-1} (ω), what must be the radius of the rotor?



$$F_f = F_g$$

$$\mu R = mg$$

$$(0.06)R = (75)(9.81)$$

$$R = 12{,}263 \text{ N}$$

$$F_c = R = m\omega^2 r$$

$$12{,}263 = (75)(5.2)^2 r$$

*Friction and weight are equal and opposite

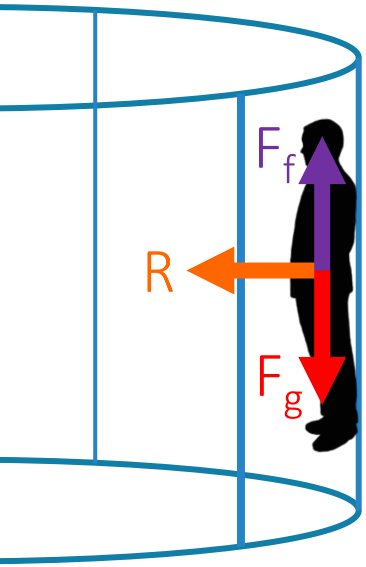
*Normal Reaction Force is equal to the Centripetal Force

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

$$r = 6.05 \text{ m}$$

You didn't need the mass ☺

The "Rotor" ride is the one which presses you against the walls of the spinning rotor as the floor drops away. The coefficient of static friction between the wall and the 75-kg rider is μ = 0.06. If the ride is rotating at an angular velocity of 5.2 rad s^{-1}, what must be the radius of the rotor?



$$F_f = \mu R$$

$$F_g = mg$$

$$F_c = m\omega^2 r$$

$$F_c = F_{net} = R$$

$$F_f = F_g$$

$$\mu R = mg$$

$$\mu \cancel{m}\omega^2 r = \cancel{m}g$$

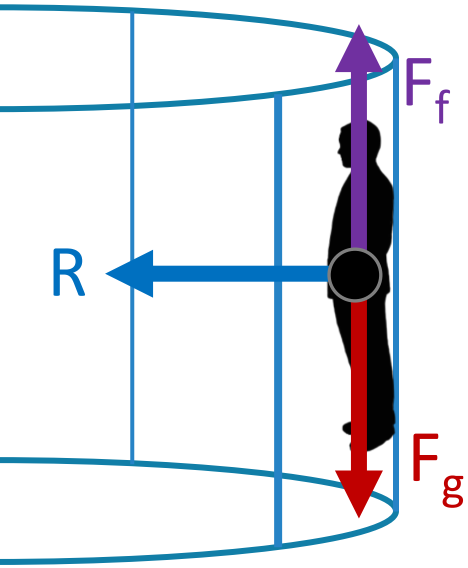
$$\mu\omega^2 r = g$$

solve for r

All Together Now!

$$F_f = F_g$$

$$F_c = R$$



Lesson Takeaways

- ❑ I can draw a free body diagram and solve a problem when circular motion is produced by a **normal reaction force**

Circular Motion Scenarios
The Curve

IB PHYSICS | CIRCULAR MOTION

IB Physics Data Booklet

Sub-topic 6.1 – Circular motion

$$v = \omega r$$

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$F = \frac{mv^2}{r} = m\omega^2 r$$

v – linear velocity (m s^{-1})

ω – angular velocity (rad s^{-1})

r – radius (m)

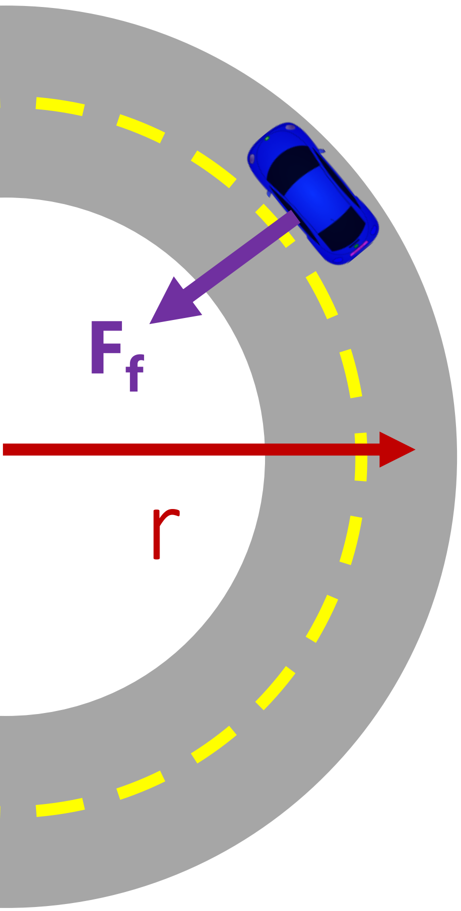
T – period (s)

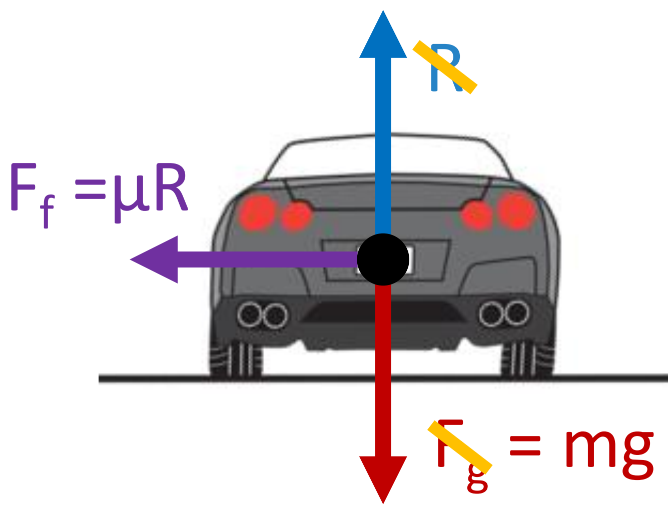
a – centripetal acceleration (m s^{-2})

F – centripetal force (N)

Skidding Around a Curve

What is providing the centripetal force causing the car to move around the curve?





Friction

$$F_{net} = F_c = F_f$$

$$R = F_g$$

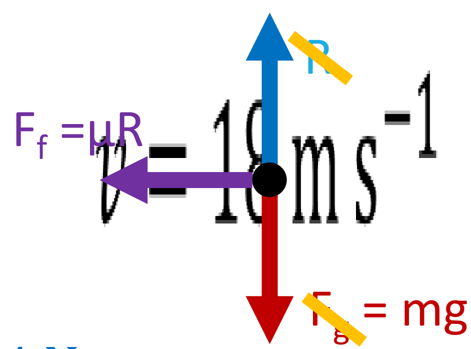
Skidding Around a Curve

A car of mass 1240 kg moves around a bend of radius 63 m on a horizontal road at a speed of 18 m s^{-1}. If the car was to be driven any faster there would not be enough friction and it would begin to skid.

What is the coefficient of friction between the road and the tires?

$$m = 1240\ \text{kg}$$
$$r = 63\ \text{m}$$
$$v = 18\ \text{m s}^{-1}$$

$$F_{net} = F_c = F_f$$



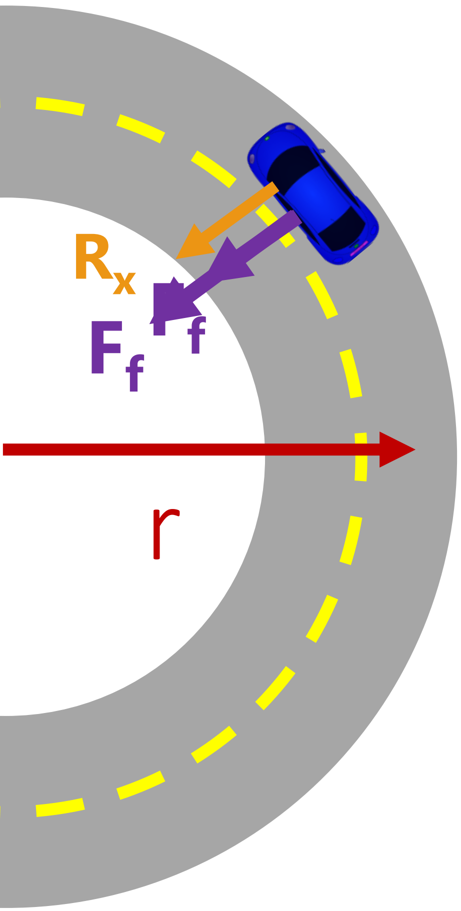
$$R = F_g = mg$$
$$= (1240)(9.81) = 12164\ \text{N}$$

$$\frac{mv^2}{r} = \mu R$$

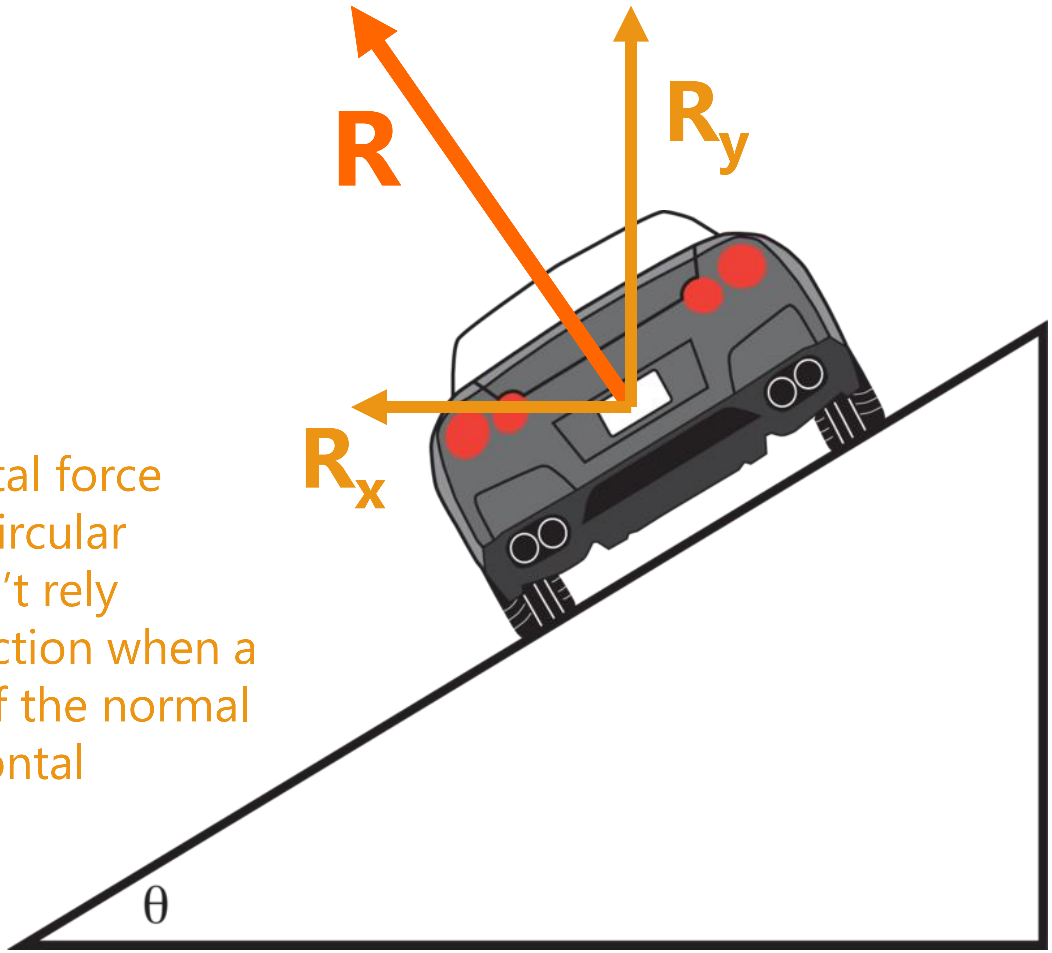
$$\frac{(1240)(18)^2}{63} = \mu(12{,}164)$$

$$\mu = 0.52$$

Banked Curve



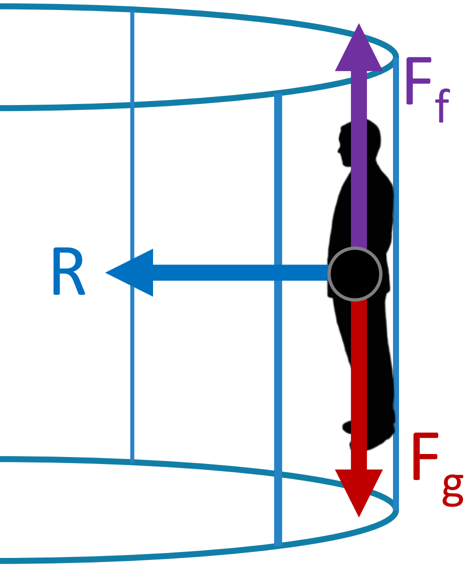
*The centripetal force creating the circular motion doesn't rely entirely on friction when a component of the normal force is horizontal

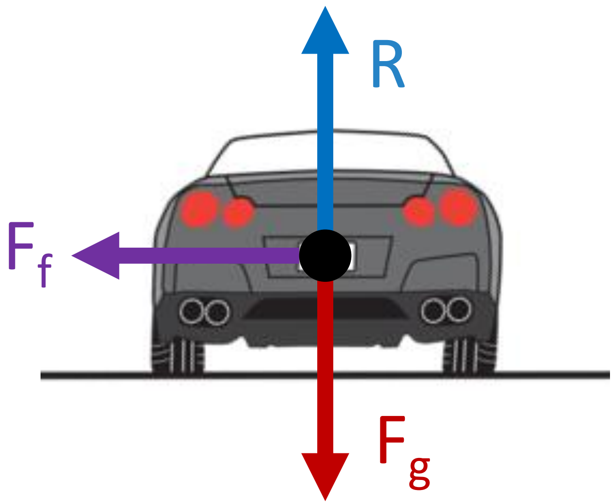


All Together Now!

$$F_f = F_g \qquad R = F_g$$

$$F_c = R \qquad F_c = F_f$$





Lesson Takeaways

- ❑ I can draw a free body diagram and solve a problem when circular motion is produced by a **friction force**

Circular Motion Scenarios
The Pendulum

IB PHYSICS | CIRCULAR MOTION

IB Physics Data Booklet

Sub-topic 6.1 – Circular motion

$v = \omega r$

$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$F = \frac{mv^2}{r} = m\omega^2 r$

v – linear velocity (m s^{-1})

ω – angular velocity (rad s^{-1})

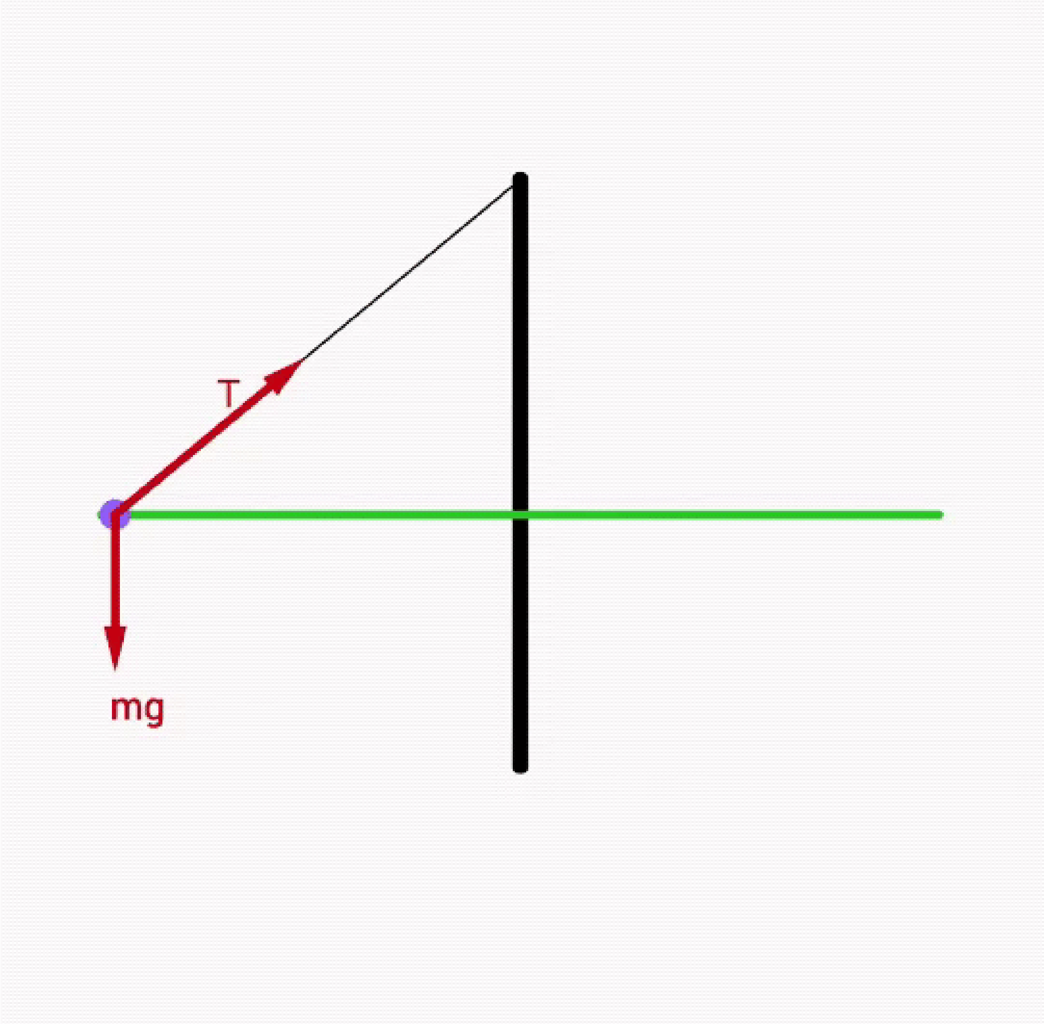
r – radius (m)

T – period (s)

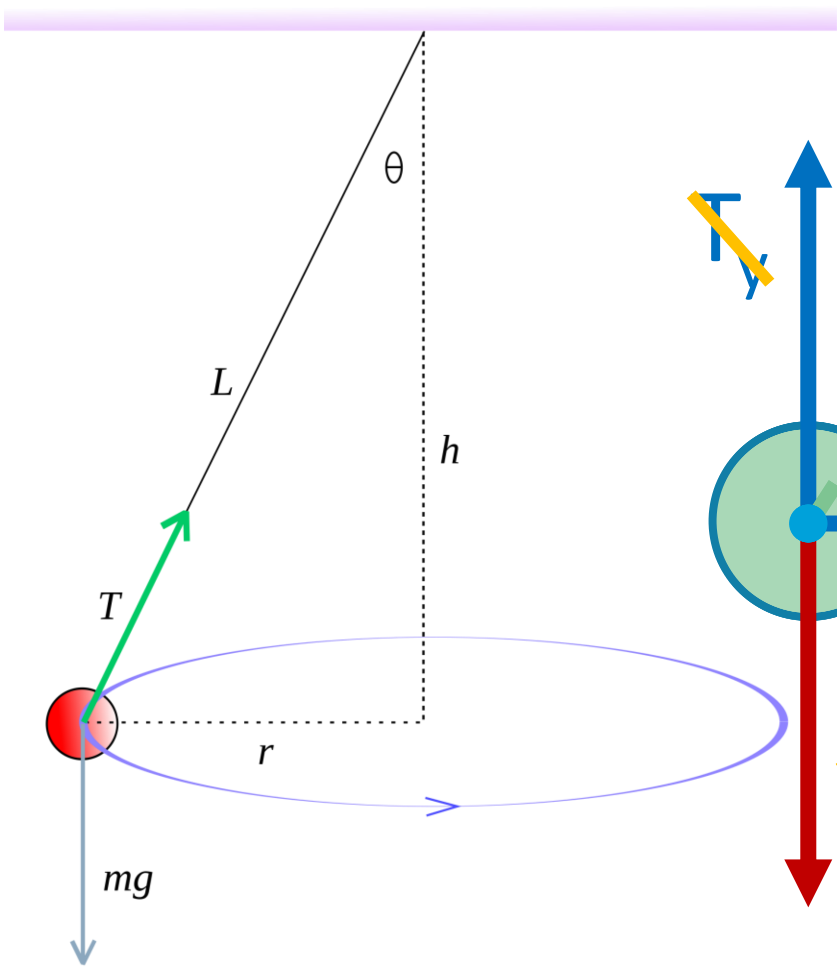
a – centripetal acceleration (m s^{-2})

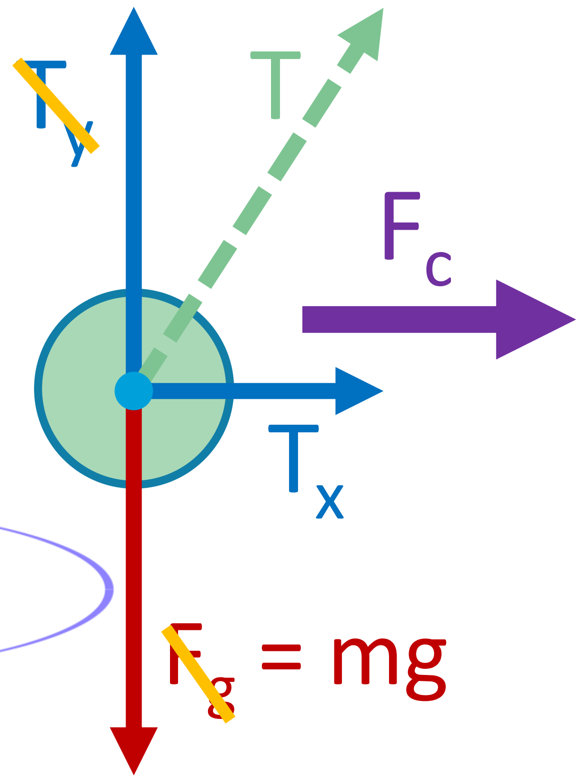
F – centripetal force (N)

Pendulum Circle



Pendulum Circle

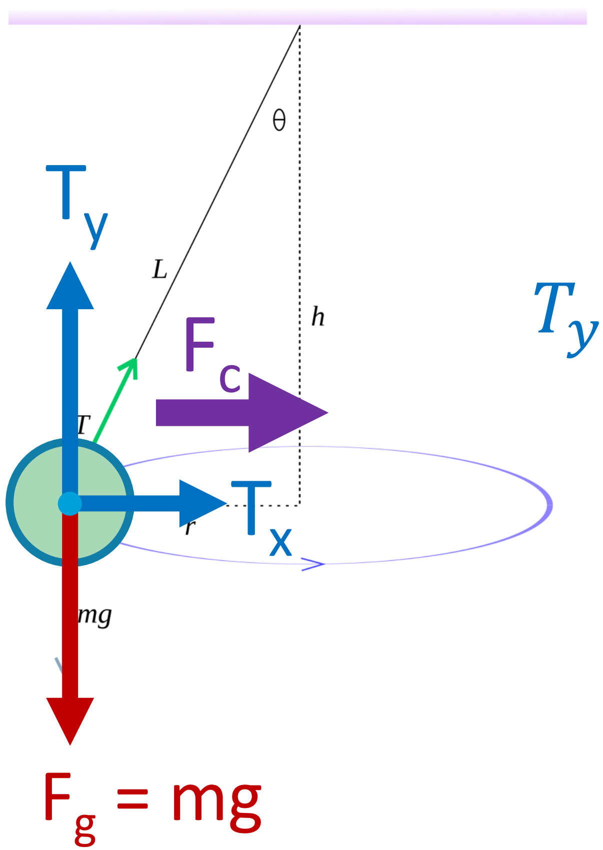




$$F_{net} = F_c = T_x$$

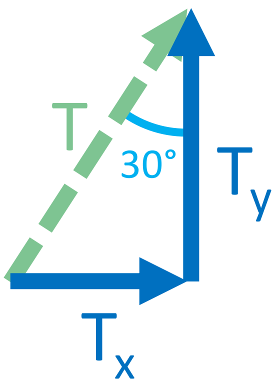
$$T_y = F_g$$

Pendulum Circle



What is centripetal force required to cause a 0.12 kg mass to swing in a horizontal circle with the string at an angle of 30°?

$$T_y = F_g = mg = (0.12)(9.81) = 1.18\text{ N}$$



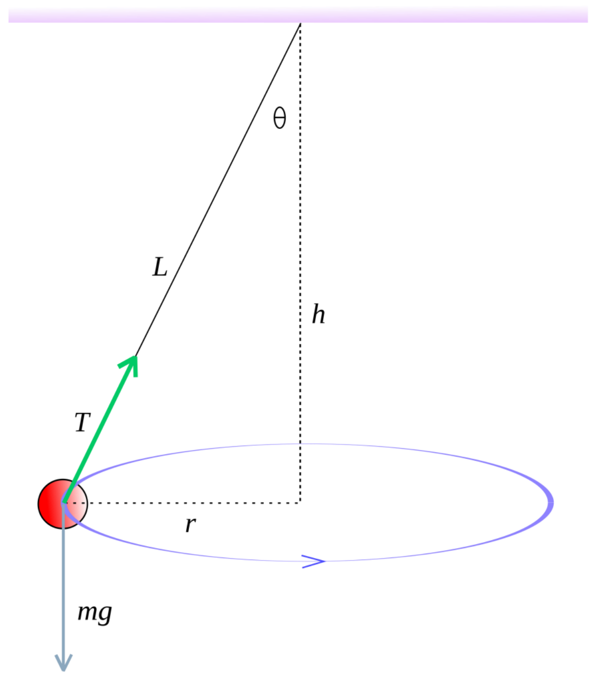
$$\tan(\theta) = \frac{T_x}{T_y} \qquad T_x = T_y \tan(\theta)$$

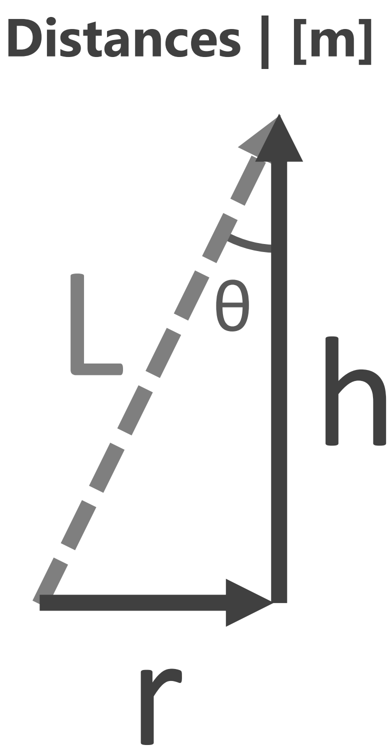
$$T_x = 1.18\tan(30°) = 0.68\text{ N}$$

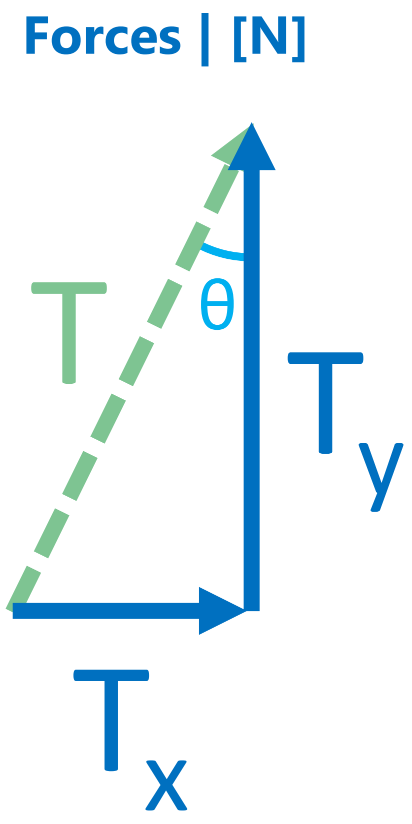
$$F_c = 0.68\text{ N}$$

$$F_{net} = F_c = T_x = 0.68\ N$$

CAUTION! There are **two** triangles



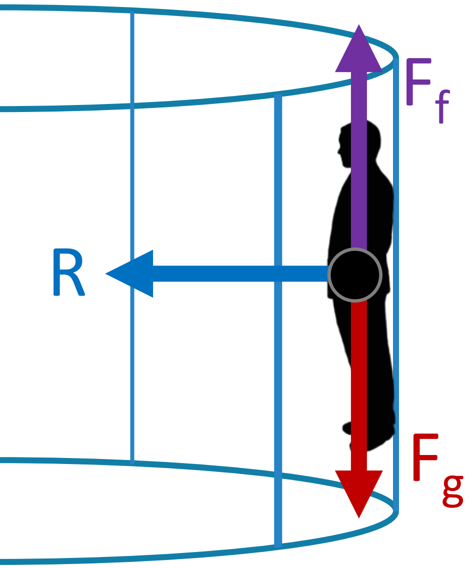


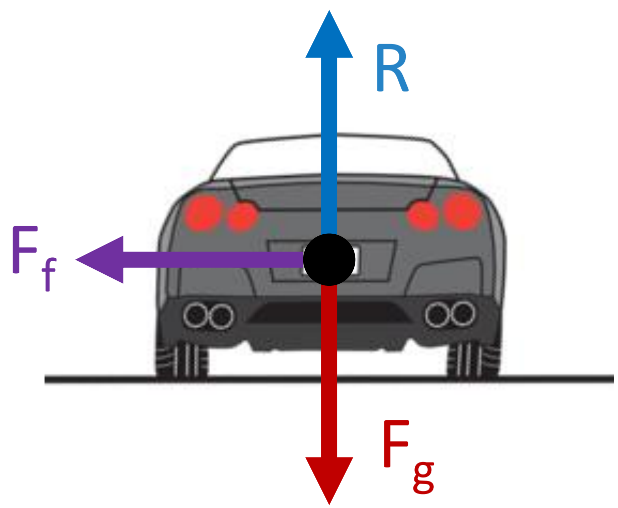


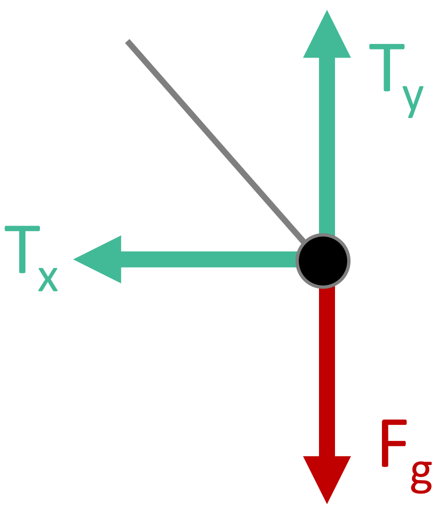
All Together Now!

$$F_f = F_g \qquad R = F_g \qquad T_y = F_g$$

$$F_c = R \qquad F_c = F_f \qquad F_c = T_x$$







Lesson Takeaways

- ❑ I can draw a free body diagram and solve a problem when circular motion is produced by **components of an angled tension force**.