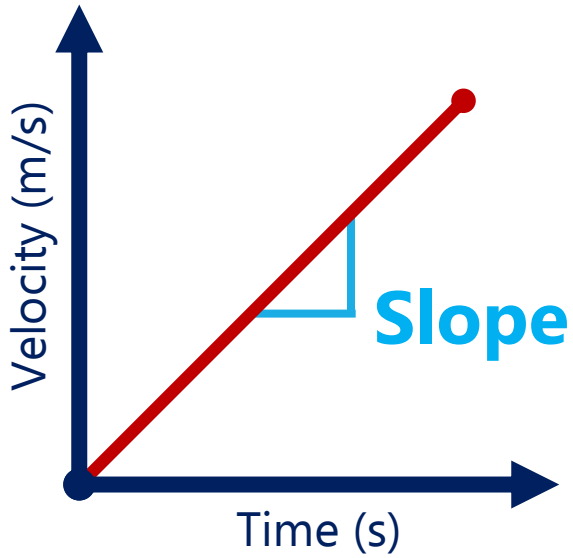
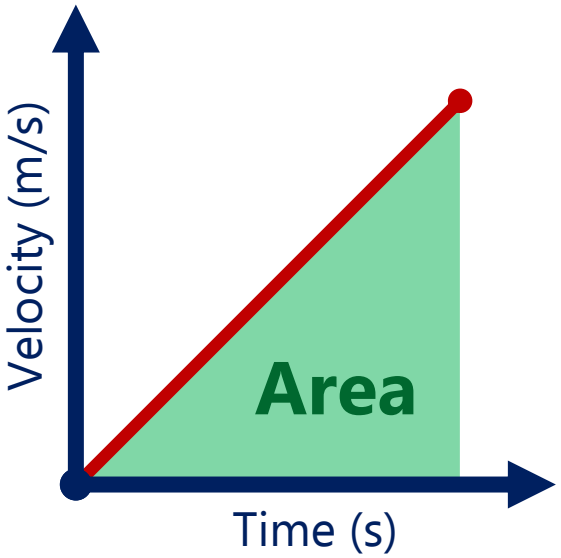
Elastic Potential Energy

IB PHYSICS | ENERGY & MOMENTUM

Calculating from a Graph





$$\frac{\Delta y}{\Delta x} = \frac{Velocity}{Time} = \frac{m/s}{s} = {}^{m}/_{s^2}$$

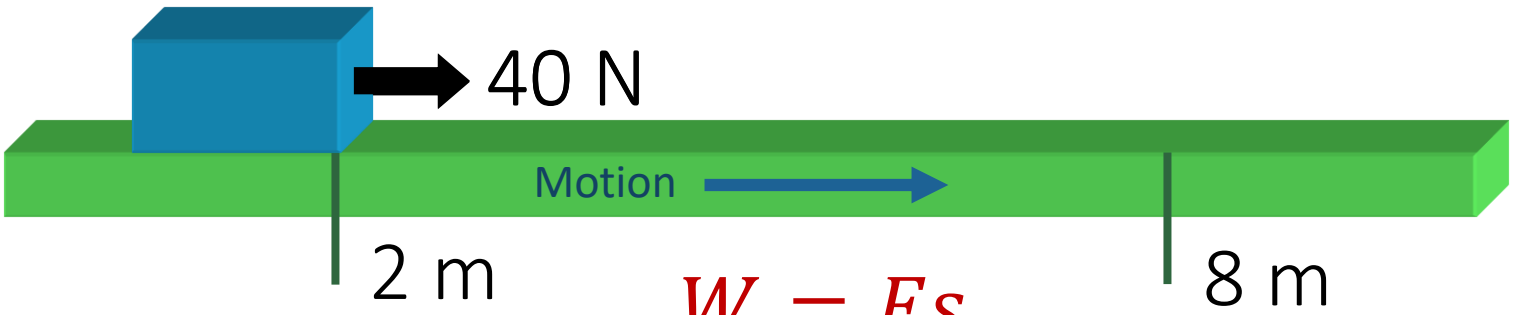
Acceleration

$$(x)(y) = (Velocity)(Time)$$

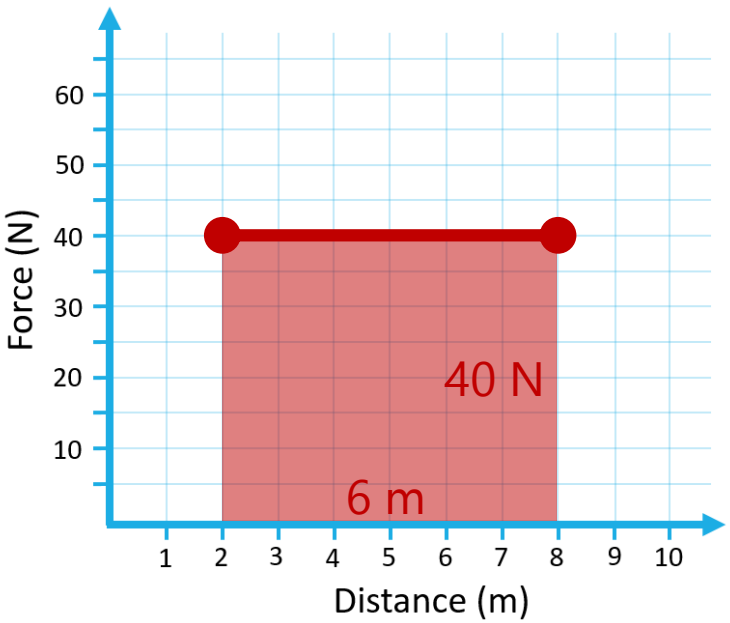
$$= \left(\frac{m}{s}\right)(s) = m$$

Displacement

Graph of Force vs Displacement



$$W = Fs$$



$$Area = (6\ \mathrm{m})(40\ \mathrm{N})$$

$$= 240\ \mathrm{Nm}$$

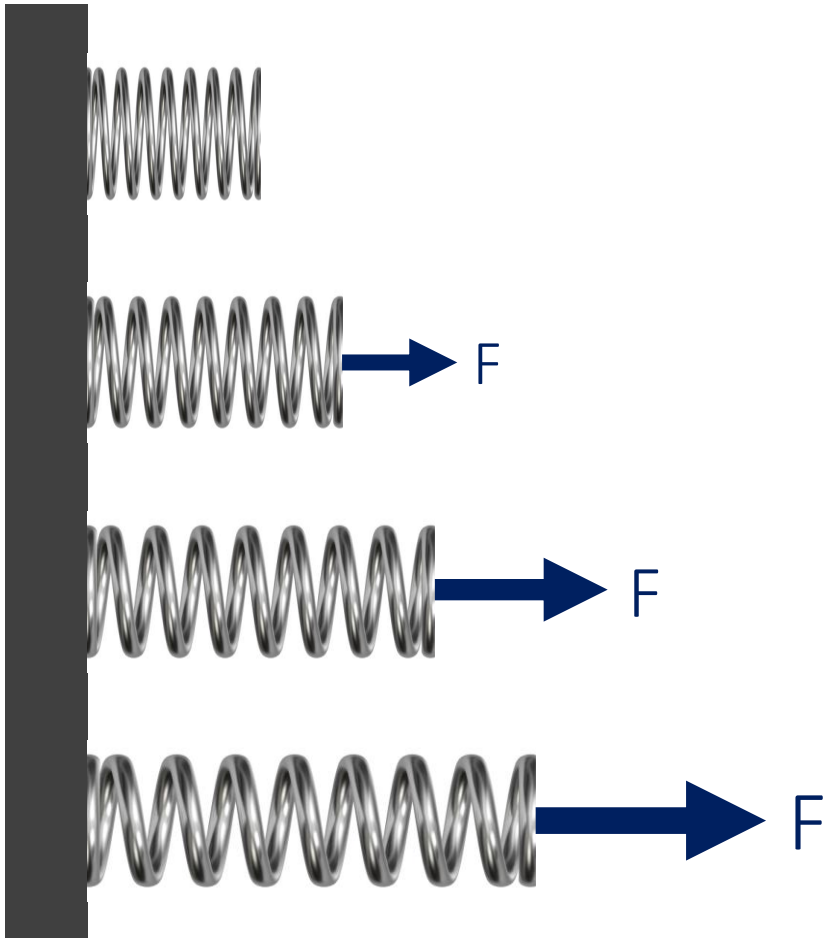
$$= \mathbf{240\ J}$$

Work of a Varying Force

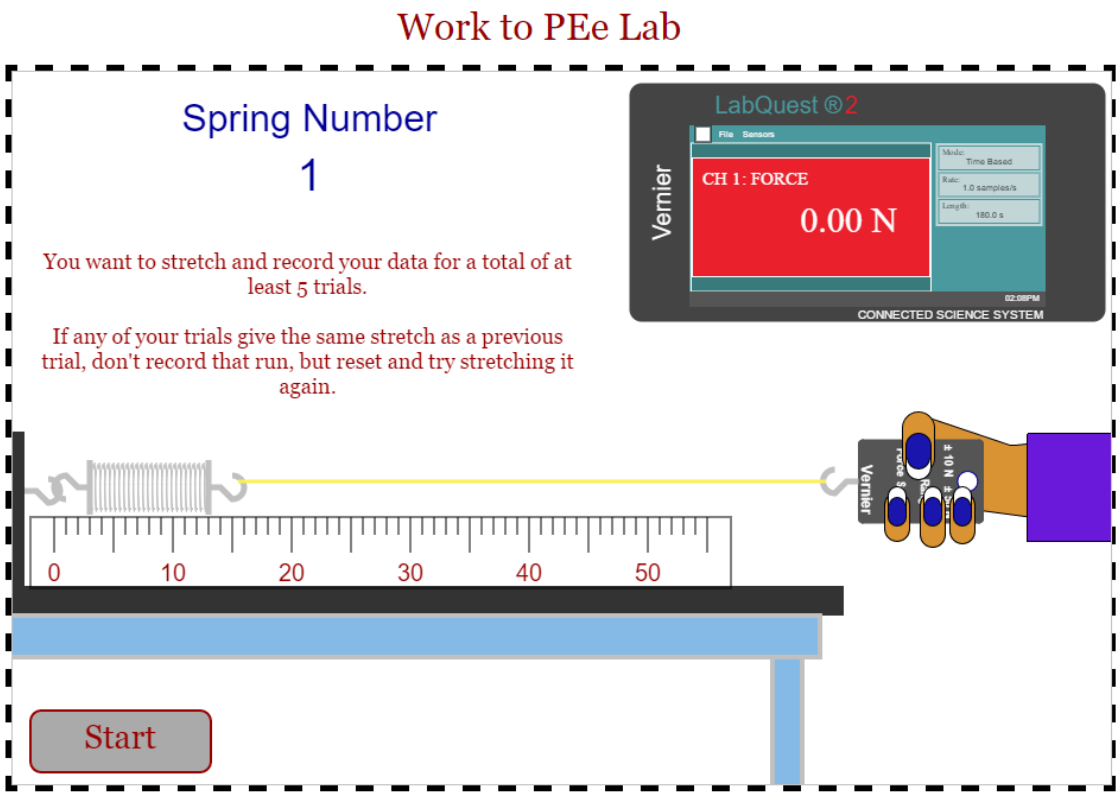
Our definition of work applies only for a <u>constant</u> force or an <u>average</u> force.

$$W = Fs\cos\theta$$

What if the force <u>varies</u> with displacement as with stretching a spring or rubber band?



What about a Varying Force?



Click here for the simulation

Work of a Varying Force

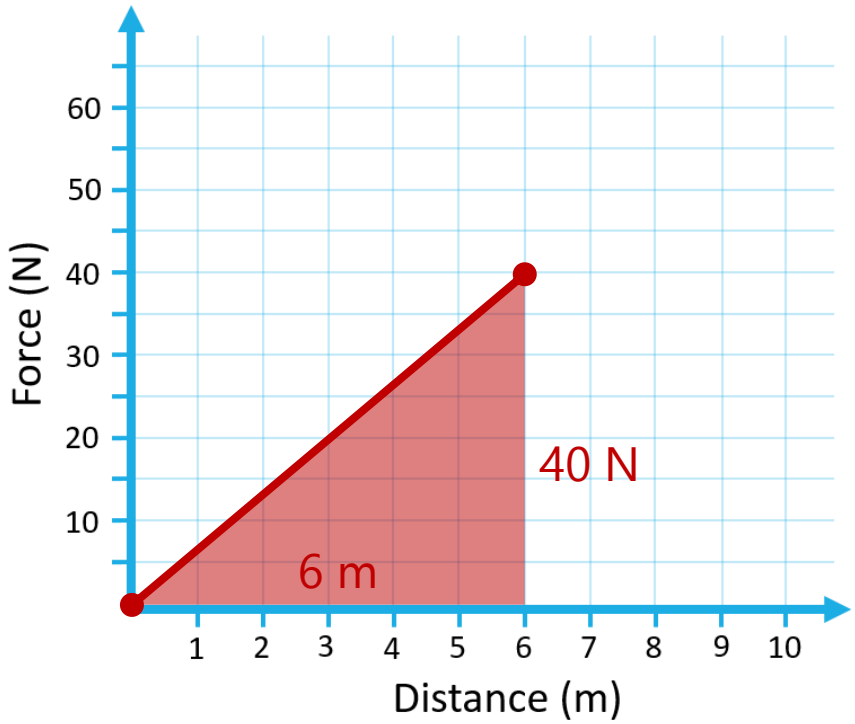
How can you calculate the work?

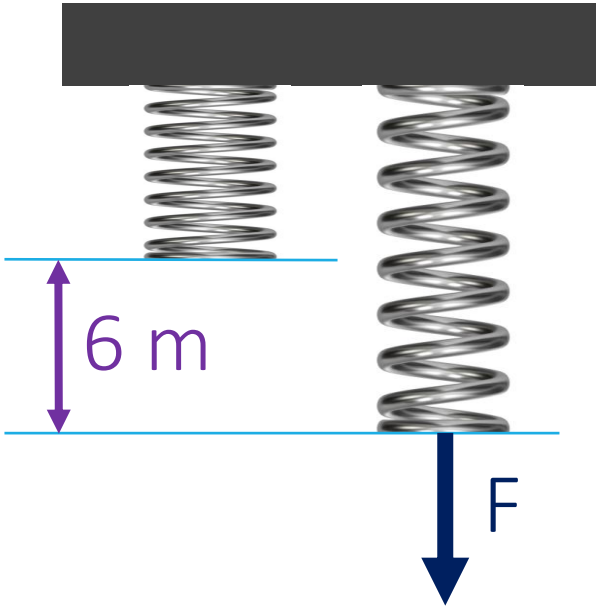
$$Area = \frac{1}{2}(10\ \mathrm{m})(40\ \mathrm{N})$$

$$= 200\ \mathrm{J}$$

Work of a Varying Force

What **work** is required to stretch this spring from x = 0 to x = 6 m?

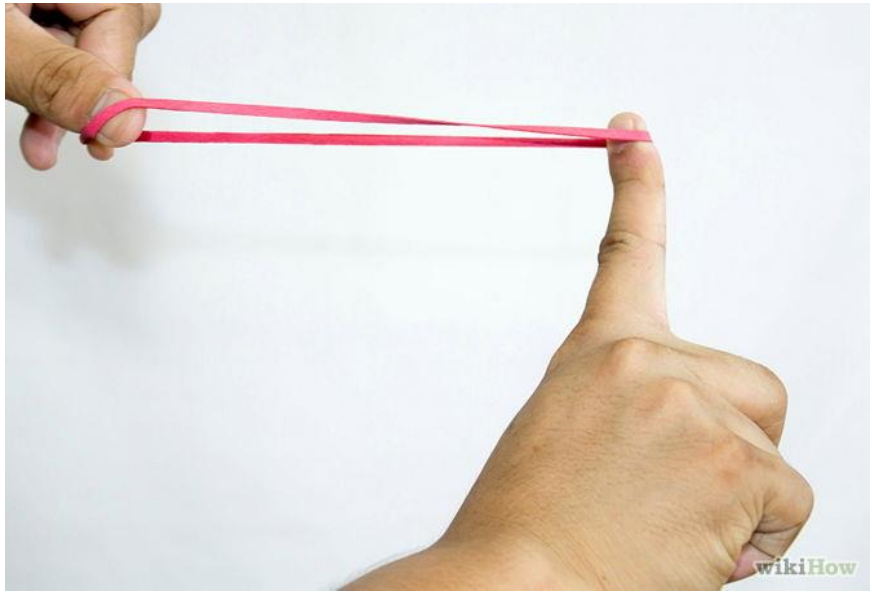




$$Work = \frac{1}{2}(6\ \mathrm{m})(40\ \mathrm{N})$$

$$= 120\ \mathrm{J}$$

Elastic Potential Energy



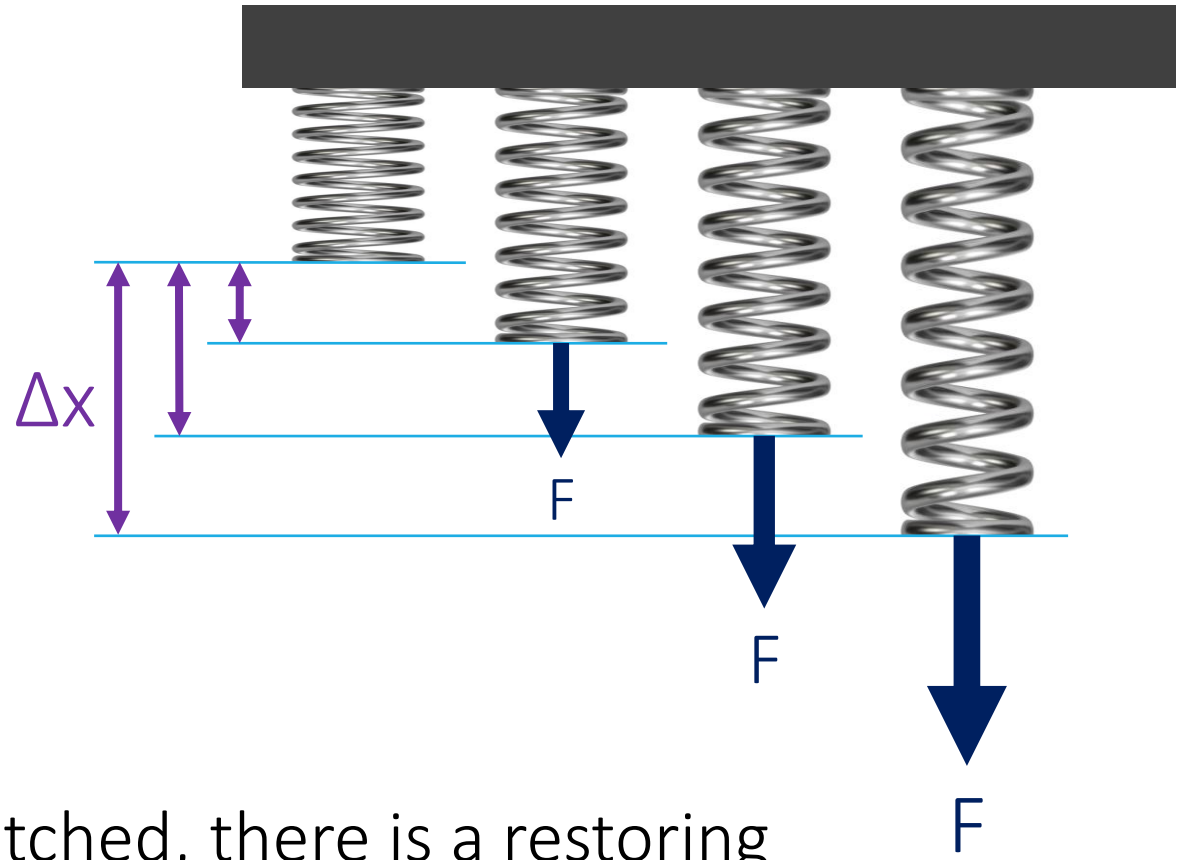
As the pull back distance increases elastic potential energy ___increases___

Hooke's Law

$$F = k\Delta x$$

**The spring constant k is a property of the spring*

$k \rightarrow [N\ m^{-1}]$



When a spring is stretched, there is a restoring force that is proportional to the displacement.

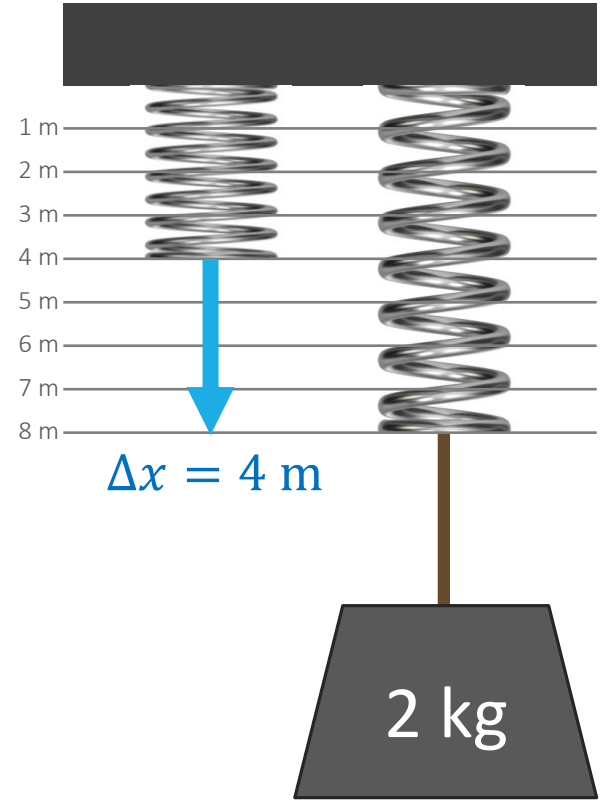
Try this…

A block with a mass of 2 kg is suspended from a spring and produces the displacement shown. What is the spring constant?

$$F = k\Delta x$$

$$(19.62\ \mathrm{N}) = k(4\ \mathrm{m})$$

$$\boxed{\boldsymbol{k = 4.9\ \mathrm{Nm}^{-1}}}$$



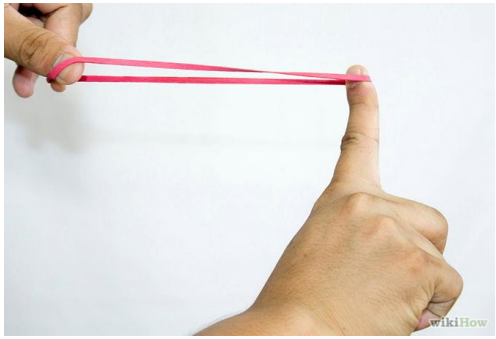
$$F_g = mg = (2)(9.81)$$

$$F_g = 19.62\ \mathrm{N}$$

Work and Energy

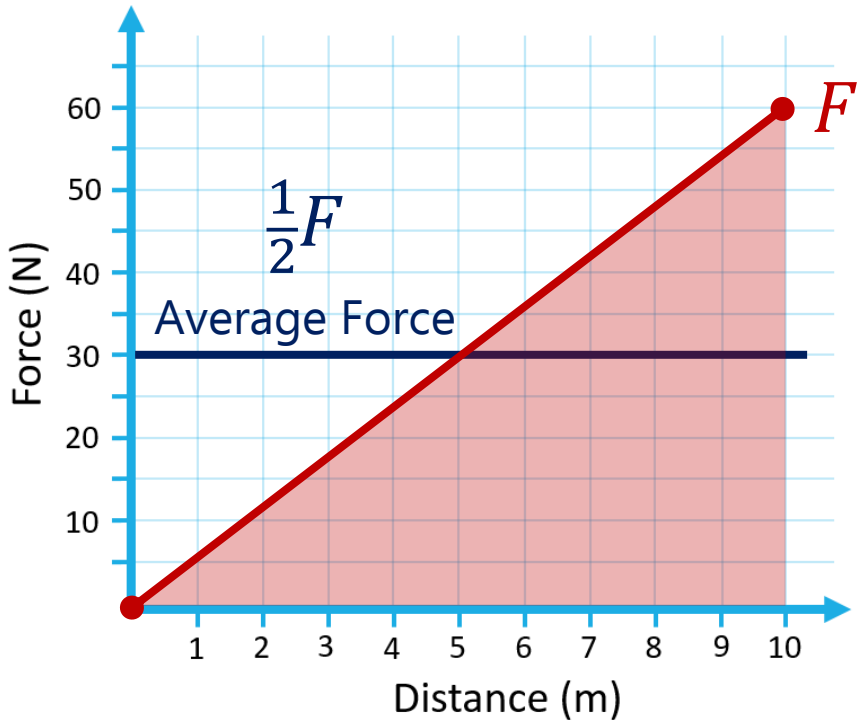
Work done on a system causes the system to gain or lose energy

Stretching or compressing a spring **stores energy**



Work of a Varying Force

Now that we know that F = kΔx, we can calculate the stored elastic potential energy with the work equation



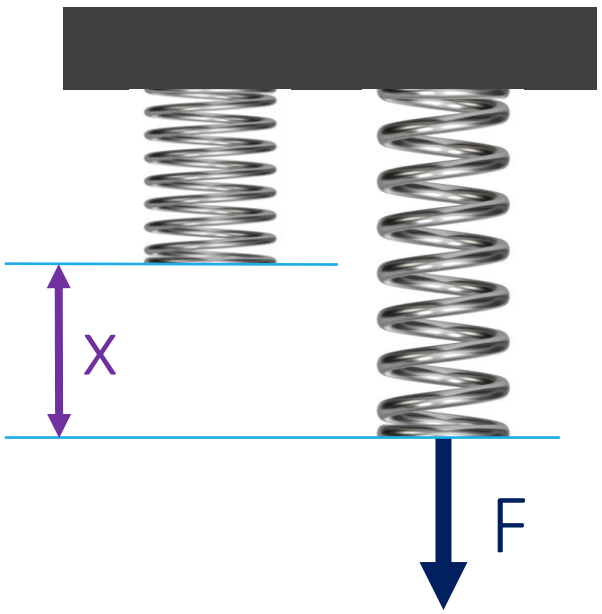
$$Work = Fs$$

$$= \left(\frac{1}{2}F\right)s$$

$$= \left(\frac{1}{2}k\Delta x\right)\Delta x$$

Elastic Potential $= \boldsymbol{\frac{1}{2}k\Delta x^2}$

Elastic Force and Work



$$F = k\Delta x$$

$$E_p = \frac{1}{2}k\Delta x^2$$

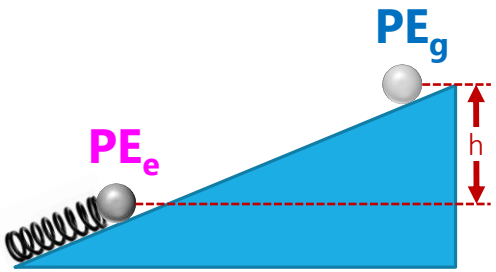
**The spring constant k is a property of the spring*

Data Booklet

Sub-topic 2.3 – Work, energy and power
$W = Fs\cos\theta$
$E_K = \frac{1}{2}mv^2$ ← velocity KE
$E_P = \frac{1}{2}k\Delta x^2$ ← elastic PE_e
$\Delta E_P = mg\Delta h$ ← gravity PE_g
power $= Fv$
$\text{Efficiency} = \frac{\text{useful work out}}{\text{total work in}}$
$= \frac{\text{useful power out}}{\text{total power in}}$

Conservation of Energy

How far up the 15° incline of a pinball table will a 0.1 kg pinball move after it is launched? The spring constant is 100 N/m and is compressed by 0.08 m.



$$\cancel{KE_{initial}} + PE_e + \cancel{PE_{g\,initial}} = \cancel{KE_{final}} + \cancel{PE_{e\,final}} + PE_g$$

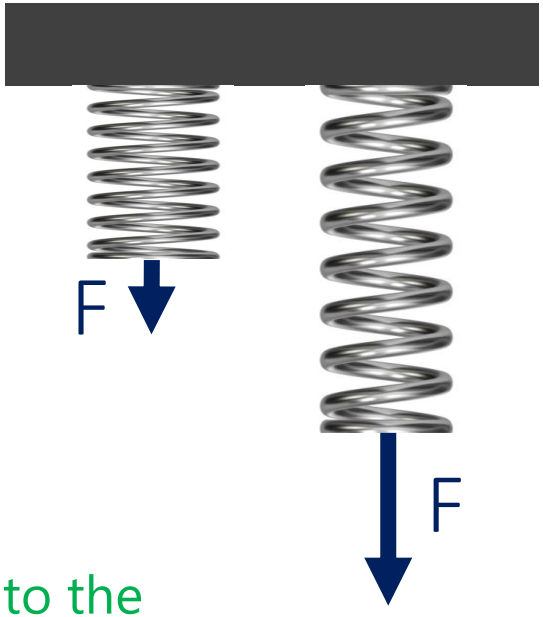
$$\frac{1}{2}k\Delta x^2 = mgh$$

$$\frac{1}{2}(100)(0.08)^2 = (0.1)(9.81)(h)$$

$$\boxed{h = 0.33\ m}$$

Try this…

What **work** is required to stretch this spring ($k = 200\ \mathrm{N\ m^{-1}}$) from $\Delta x = 0.1$ m to $\Delta x = 0.4$ m?



Initial

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}(200)(0.1)^2 = 1\ \mathrm{J}$$

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}(200)(0.4)^2 = 16\ \mathrm{J}$$

Final

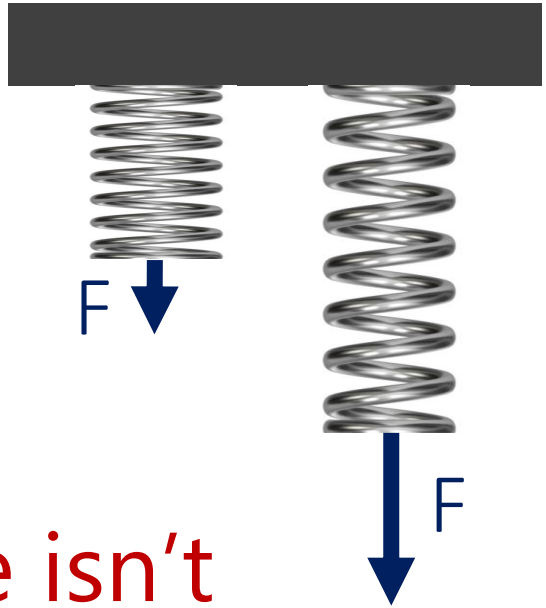
15 J

Added to the system through work on spring

$$\boldsymbol{W = 15\ \mathrm{J}}$$

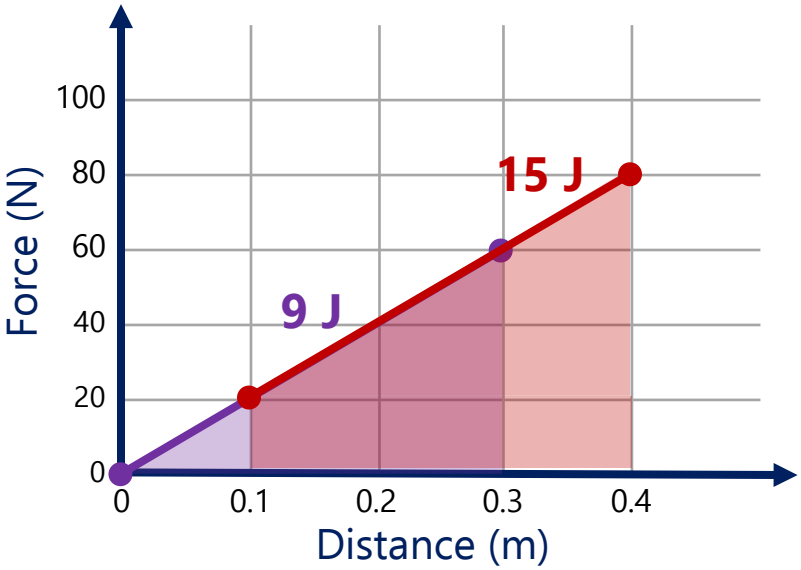
Try this…

What **work** is required to stretch this spring ($k = 200\ \mathrm{N\ m^{-1}}$) from $\Delta x = 0.1$ m to $\Delta x = 0.4$ m?



Why not just use the stretch change?

$$\cancel{\frac{1}{2}k\Delta x^2 = \frac{1}{2}(200)(0.3)^2 = 9\ \mathrm{J}}$$



The force isn't starting at zero!

$$\boxed{W = 15\ \mathrm{J}}$$

Example IB Question

An increasing force acts on a metal wire and the wire extends from an initial length l_0 to a new length l. The graph shows the variation of force with length for the wire. The energy required to extend the wire from l_0 to l is E. The wire then contracts to half its original extension. What is the work done by the wire as it contracts?

A. 0.25*E*

B. 0.50*E*

C. 0.75*E*

D. E

$E - E_{new}$

$E - 0.25E$

$0.75E$

Lesson Takeaways

- ❑ I can calculate work as area bounded by a Force vs Distance graph
- ❑ I can use Hooke's Law to calculate the elastic force at a given displacement
- ❑ I can describe and calculate elastic potential energy